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GAZETTE

EDITED BY

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WITH THE CO-OPERATION OF

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AND

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Change of Address should be notified to a Secretary. If Copies of the "Gazette"
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The Mathematical Association.

THE Annual Meeting of the Mathematical Association was held at the London Day Training College, Southampton Row, London, W.C. 1, on Monday, 7th January, 1924, at 5.30 p.m., and Tuesday, 8th January, at 10. a.m. and 2.30 p.m. Professor Alfred Lodge took the chair in the absence of the President by reason of illness.

MONDAY, 5.30 p.m.

(1) " Earthquakes," by Professor H. H. Turner, M.A., D.Sc., F.R.S.

TUESDAY MORNING, 10 p.m.

BUSINESS.

(2) The following Report of the Council for the year 1923 was distributed and taken as read :

DURING the year 1923, 77 new members of the Association have been elected, and the number of members now on the Roll is 934. Of these, 7 are honorary members, 52 are life members by composition, 24 are life members under the old rule, and 851 are ordinary members. The number of Associates connected with the Local Branches of the Association is about 500.

The Council regret to have to record the deaths of the Rev. A. J. C. Allen, formerly Fellow of Peterhouse, Cambridge, and a member of the Association for more than forty-one years ; Mr. G. Gledhill, of Walsall ; Professor G. B. Halsted, Ph.D., of Colorado, U.S.A., a member of the Association since 1892 ; Professor T. P. Kent, of the University of Cape Town ; and Mr. Rawdon Levett. Mr. Levett was the first Honorary Secretary of the Association, and held that office from the foundation of the Association in 1871 until 1882. An

obituary notice of Mr. Levett appeared in the *Mathematical Gazette* for July, 1923.

Early in the year 1923 the General Teaching Committee, the Boys' Schools Committee, and the Girls' Schools Committee were reconstituted in accordance with the new scheme.   The Committees have since been engaged on the consideration of reports on the " Teaching of Mechanics in Girls' Schools," on the " Teaching of Mathematics in Preparatory Schools," and on the " Teaching of Geometry in Schools."   Two of these have already been issued to members, and the other is nearly ready.

At the November Meeting of the Council the Cardiff Mathematical Society applied for affiliation to the Association, and the Council had much pleasure in granting the application.   Mr. A. T. Wadley, of University College, Cardiff, is Honorary Secretary of the Society. Eight such bodies are now affiliated to the Association.

The Library of the Association is now housed at 160 Castle Hill, Reading, and Professor E. H. Neville has taken up the duties of Librarian.   The advantage to members is shewn by the fact that in the six months following the removal the number of loans was as great as in the four years preceding.   Reasons for the increase are : (1) Prof. Neville has been able to acquire sufficient familiarity with the books to respond to enquiries of the vague form, " Have you any book dealing with so-and-so ? " and to recognise (the case is imaginary) Robertson's *Analytical Conics* in " A book on coordinate geometry by Richardson ; " (2) A considerable proportion of the loans have been private loans, negotiated for members with owners of books not in the Library ; (3) The generosity of donors, by eliciting acknowledgments in the *Gazette*, has kept the Library prominently before members of the Association.

The Library has been greatly enriched during the year ; the principal benefactors have been Mr. Greenstreet and the Rev. J. J. Milne.

The question of again holding a summer meeting, away from London, has been raised.   The Council would be glad to have the views of members on this matter.   The meeting held at Leeds in May, 1920, was very successful.

Sir Thomas L. Heath retires at this meeting from the office of President, and the Council desire to express their deep sense of the valuable services which he has rendered to the Association during his two years of office.   They feel that their gratitude to the retiring President will be shared by every member of the Association.   A permanent memento of his term of office remains in copies of his three-volume edition of Euclid's *Elements* and his two-volume *History of Greek Mathematics* which he has generously given to the Library.

The Council have the pleasure of nominating Professor G. H. Hardy, M.A., F.R.S., Savilian Professor of Geometry in the

University of Oxford, to be President of the Association for the years 1924 and 1925, and Sir Thomas Heath to be a Vice-President.

The Council have also the pleasure of proposing that Mr. W. J. Greenstreet be elected an Honorary Member of the Association in commemoration of the completion of twenty-five years of his editorship of the *Mathematical Gazette*, and in recognition of the many and great services he has rendered to the Association.

Mr. R. C. Fawdry has expressed a desire to resign his seat on the Council and Mr. W. E. Paterson retires by rotation. They are not eligible for re-election for the coming year. The members present at the Annual Meeting will be asked to nominate and elect others to fill these vacancies.

The Council again desire to acknowledge the indebtedness of the Association to Mr. W. J. Greenstreet for his services as Editor of the *Mathematical Gazette*.

(3) The Treasurer briefly indicated the nature of the Report for the year 1923.

(4) Professor E. H. Neville, Mr. F. G. Hall, and Miss Punnett reported on the work of the Teaching Committees in the year 1923.

(5) The Election of Officers and Members of Council for the year 1924 was proceeded with.

Mr. C. O. Tuckey, Charterhouse School, and Miss E. R. Gwatkins, Streatham Hill High School, were elected Members of Council in place of Mr. R. C. Fawdry, who had resigned, and of Mr. W. E. Paterson, retiring by rotation and ineligible for re-election, 1924.

(6) The nomination by the Council of Professor G. H. Hardy, F.R.S., Savilian Professor of Geometry in the University of Oxford, to be President of the Association for the years 1924 and 1925 was received with applause, and the nomination of Sir Thomas L. Heath, K.C.B., to be a Vice-President was also cordially endorsed.

(7) Mr. W. J. Greenstreet, on the proposal of the Council, was duly elected an Honorary Member of the Association on the grounds set forth in the Report above.

(8) Mr. W. C. Fletcher delivered an address on " English and Mathematics." Among the speakers who took part in the discussion were Professors Steggall and Godfrey, and Mr. R. C. Fawdry.

(9) Mr. W. Hope-Jones read a paper entitled " A Plea for Teaching Probability in Schools."

Lunch Intervened.

(10) Mr. A. W. Lucy exhibited a Surveying Instrument and explained its use in connection with Practical Trigonometry.

(11) Owing to the absence through illness of the President, Sir T. L. Heath, K.C.B., the Presidential Address was not delivered, but a letter from the President was read by the Chairman.

(12) Discussion on the Report on the Teaching of Geometry. Professor E. H. Neville, Chairman of the Sub-Committee that drew up the Report, opened the discussion, in which took part Messrs. W. C. Fletcher, J. Katz, A. A. Bourne, C. W. Tregenza, S. Inman, E. Montague Jones, and Dr. W. F. Sheppard.

(13) Mr. G. Goodwill read a paper on " Euclid and his Successors : some confusion and a way out."

(14) Professor C. Godfrey, M.V.O., gave an address entitled " Constructions in Geometry. What is legitimate ? "

QUEENSLAND BRANCH OF THE MATHEMATICAL ASSOCIATION.
FIRST ANNUAL REPORT, 1922-23.

ON March 31st, 1922, at the University, a General Meeting was called by Professor H. J. Priestley, M.A., to receive the report of a sub-committee appointed at a previous meeting.

The report as read was adopted unanimously as a Constitution for the Queensland Society, and an election of officers at once took place.

On May 5th, 1922, Professor H. J. Priestley, M.A. addressed the Society at the Boys' Grammar School, Gregory Terrace, on " Some Problems in Mathematical Education." A discussion ensued.

The Society held a very successful dinner at the Hotel Cecil on Thursday evening, 15th June.

The second General Meeting was held at the University. Mr. S. Stephenson, M.A., gave a very instructive address on " Geometry."

On October 20th the Society held its third meeting at the Boys' Grammar School, Ipswich. Mr. and Mrs. R. A. Kerr very kindly entertained the visitors at tea, after which Mr. R. A. Kerr, M.A., addressed the meeting, dealing mainly with the teaching of Arithmetic in the Schools. A discussion followed.

During the year four committee meetings were held, at which arrangements for the work of the Society were made.

The Secretary was asked to circularise the Secondary Schools of the State, pointing out the reasons for the formation of the Queensland Branch of the Mathematical Association, and asking for co-operation. The response has been rather disappointing. Out of the 28 members of our Society, only six belong to country schools. It may be mentioned that 49 schools were circularised, and of the six country members three belong to one school.

Nine members have become full members of the Mathematical Association, London.

Copies of the *Gazette* for July, October, and December, 1922, were sent gratis to our Society, the Secretary of the Mathematical Association agreeing to reckon our subscriptions as from the 1st January, 1923.

At a committee meeting, October 20th, 1922, it was decided, owing to the delay in obtaining literature during 1922, that outside members of the Society should be free from subscription for the year 1923-4.

The finances of the Society are in a healthy state, the balance sheet showing a small credit balance of 16s. 10d.

The first year of the Society has been very successful, and with the continued support of the members, we can look forward to greater activities and scope for the Society in 1924.   STANLEY G. BROWN,
6.4.'23.   *Hon. Secy. and Treas.*

I.

ENGLISH AND MATHEMATICS.

By W. C. Fletcher, M.A.

The Report on " The Teaching of English in England " lays great stress on the important truth that the use of English concerns all teachers, but when it deals with the matter more closely it enumerates teachers of history, languages, geography and science, and says nothing about mathematics (*e.g.* § 110).

If this fact stood alone, little need be said ; but there is ample evidence that teachers of mathematics themselves insufficiently realise that the dictum applies to them quite as much as to other teachers.

It seems, therefore, desirable to consider with some care the relations between mathematics and language with a view to establish what seems to the writer the two-edged truth :

(*a*) The study of mathematics has contributions of great, even of unique, importance to make towards training in the use of English ;

(*b*) It is impossible to teach mathematics properly unless these contributions are made.

Quotations from the Report will help to define the issues and facilitate discussion.

" The premature introduction of . . . Latin and Mathematics not only encroaches on the time needed for English, but has a definitely injurious effect on the mind . . . difficult subjects the rationale of which is quite beyond his ken " (§ 99).

The premature introduction of any subject is, of course, mischievous and it must be admitted that mathematicians have erred grievously in this respect ; but they are not alone in their error and teachers of English themselves not infrequently do serious harm by asking for literary appreciation and criticism far too early.

But is mathematics a difficult subject ?   Absolutely, of course it is, as is every other subject when the aim is high ; perfection, even excellence, in any pursuit is difficult.   But when people say that mathematics is difficult, they generally mean more difficult than other subjects, and the common opinion probably is that mathematics is the hardest, English the easiest subject of the curriculum.

This seems to be an exact inversion of the real truth : to establish this fact and to lay bare the reasons for the misconception is the object of the following paragraphs.

Consider first the way in which mathematics enters into the affairs of life. A man has money to invest ; he may have difficulties in the mere arithmetic : how much of a given stock at a given price can he buy, what will be the subsidiary expenses and what income will he derive ?   But there are far greater difficulties to be faced before he comes to the arithmetic : can he trust any foreign government, or even his own ?   Are railways safe, or will industrial troubles bring them to ruin ?

Perhaps, again, the citizen finds it difficult to read his gas meter and to check arithmetically the demand made on him by the rate-collector.   But how much greater the difficulty of deciding as between Free Trade and Protection !

The Lords of the Admiralty during the war set their naval constructors some very difficult problems in applied mathematics, but was not their own difficulty far greater in deciding what kind of ships it was best to have ?

In appearance it is easier to read a few pages of history than a page of geometry, but is not the appearance delusive ?   Even to master the mere facts as stated is difficult to most boys, while really to understand them

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surely requires an experience of life and a power of envisaging the past which are quite beyond his scope.

The teacher of geography no doubt finds difficulty in making his class understand the phenomena of the seasons, day and night, as related to the motions of the earth, where the conceptions required are purely geometrical; but he finds it far harder to give them an equally adequate understanding of climate, weather, winds, where physical laws are required.

Or to speak of literature, is it really as hard to understand a proposition of Euclid, even to master it, as to fathom the meaning of a passage in Hamlet or to follow the argument in a speech of Burke's ?

Mathematics, school mathematics at least, until one comes to deal with its philosophy, is concerned and can only be concerned with the simplest things and their simplest combinations : everything is distinct, exact, certain ; it does not deal with the play of feeling and passion as do English and history ; with the reconstruction of a forgotten past from fragmentary and contradictory materials as do history and geology ; nor even with the complexities of physical conditions as does geography.

Why, then, if this view is right, if mathematics is essentially a simple subject, has it got the reputation of being difficult ?

One reason undoubtedly is bad teaching and a bad tradition of teaching. The teachers of a generation ago commonly treated it as difficult and even made it difficult by terror, and the traditional method of dealing with geometry was fundamentally wrong. These troubles are passing away and the modern schoolboy probably does not regard the subject as difficult at all.

But there is another reason which is more or less permanent and which it behoves teachers of other subjects (English especially) to consider with some care.

In mathematics there is generally an absolute right and wrong : error can be detected with comparative ease and can then be definitely convicted as error ; very commonly indeed it convicts itself. In other subjects this is much less true ; even to find out whether a lesson in history has been tolerably learnt is a much slower and more uncertain business than to control work in mathematics. English work in particular is difficult to check ; the marking of essays is notoriously uncertain, leaving much room for differences of opinion ; while to ensure that a piece of literature is really understood (let alone ' appreciated ') is perhaps a more difficult task still.

Hence work in English, even in history, is in constant danger not merely of being loose and unsound, but of degenerating into mere pretentious humbug.

The better teachers of English are, of course, aware of this danger and take pains to meet it ; none the less the danger is real and is not always avoided.

Mathematics, though simple, is exacting and is therefore called difficult ; geography, history, and above all English, are complex and to the genuine student infinitely exacting ; but just because they are so vast and difficult, their demands are more easily shirked, and unless the teacher takes unusual pains, they are shirked and that without detection.

" The study of Mathematics or of Science depends much less than do other studies upon the constant use of language " (§§ 124, 155).

There is, of course, in Mathematics a large amount of purely formal, mechanical work in the execution of which thought in the proper sense is not concerned : mere addition or multiplication sums, algebraic manipulations and the like. Indeed, algebra proper is a mechanical means of dispensing with thought, hence its great power on the one hand and its tendency to degenerate into mere trickery on the other : it requires merely the patient following of certain simple invariable rules ; it requires close attention and a sharp eye ; it may be laborious and exhausting as is the work of a bricklayer's labourer, but its very essence is that it is mechanical and not thoughtful. With this aspect of mathematics we are not here concerned.

But mathematics no more consists of this than does history of the dates of the kings of England or classics of the jingles of the Latin Primer. Language training consists of two main elements, reading and writing, both taken in the wider sense, to include hearing and speech. The essence of reading is understanding, and the test of understanding is execution in some form or other. Now if a boy is presented with a sum, ready set down in figures, there is nothing to understand (supposing he knows the rule) : he has merely to obey simple definite orders. But if he is presented with a problem, *i.e.* a written statement, his first business is to find out the meaning of the statement and the next to express it in some other form—a very real exercise in translation.

The untrained boy breaks down at the first step; he cannot read with intelligence. It is one thing to read, quite another to read with understanding. How many people for instance gain any definite picture from a descriptive passage in a novel ? One is tempted to think sometimes that the writer himself had no real picture in his mind and is merely piling up words without effective significance. Generally speaking, it is difficult to read closely unless one has a question in one's mind for which one is seeking the answer, or unless as the result of reading, some definite action has to be taken. Now in very much of the reading a boy has to do for school purposes it is difficult to provide adequate stimulus, sanction or test to secure close reading. Even in translating, a passable version may sometimes be given and yet the translation may have been purely mechanical and the substance as well as the details of the passage translated may have left no impression whatever on the mind. How often does the ordinary teacher, after his class has gone through a chapter of Caesar, ask them to shut their books and say what the content of the chapter was ? If, again, a boy is set to prepare a few pages of history, how much does he get from them unless he is made to analyse them or carry out some other form of activity ?

These are difficult exercises and it takes both good teaching and willing co-operation on the part of the class if they are to be done properly—and unless they are the reading is barren. But in mathematics this same exercise is presented in an ineluctable form, and in all stages of difficulty, especially in the simplest. The statement of a mathematical question is brief, like a simple Latin sentence ; it generally contains no irrelevant matter, and beyond a few technical terms, which, unless by the fault of the teacher, cause no difficulty, is presented in the simplest possible form : if a boy cannot understand this, he cannot be expected to understand anything. Further, the question demands action, and action is impossible until the question has been really understood. The first attempts at understanding and consequent action may be tentative or vague : but the necessity for action compels re-reading and pulling to pieces, until by degrees the whole question is fully grasped, reordered and translated into the proper shape for execution. This is surely an exercise in reading of the highest value, elementary no doubt and vastly easier than ordinary reading in history and literature if properly carried out ; but just for that very reason—because it is elementary, because it cannot be shirked, because it demands action—of the highest importance as a means of training in close reading.

Let us take some illustrations.

The set sum : " Multiply 5934 by 475 " ; or " Construct a triangle whose sides are 3, 4 and 5 inches and measure the biggest angle " ; or " Solve the equation $(x-2)(x+3)=7$ is a mere drill exercise, making little demand on intelligence, calling only for the mechanical exercise of a rule. With this necessary side of mathematics we are not concerned.

But consider the following questions :

(1) A drawing question suitable for boys of eleven who are starting geometry: Exeter is 48 miles W. of Dorchester and Barnstaple is 35 miles N.W. of Exeter. What is the distance, and bearing of Barnstaple from Dorchester ?

If the teacher, as often happens, does the question on the blackboard first and then tells the boys to do it, he practically destroys its whole value. The essential point is to leave the boys to translate the written statement into a drawing for themselves. To the teacher who acts as above described it is apparently incredible that they can do this ; to others it may seem equally incredible that they cannot. The fact is that a considerable proportion of an ordinary class will do it quite decently unaided ; but others will make mistakes showing that they have not read the question properly ; put Dorchester W. of Exeter, confuse N.W. and N.E. or not appreciate that N.W. is an exact direction not anything vaguely between N. and W.

To discuss these probable mistakes before they are made is foolish; if they are made, the proper remedy is not to point them out or ' explain ' them, but to tell the offender to read more carefully—he has already been convicted of error by his wrong answer.

A poor teacher will spend a period over explaining and spoiling a trivial question of this sort ; a skilled teacher will get his boys doing half a dozen such questions in the time, without assistance beyond occasional criticism or perhaps a rapid examination of some technical point.

(2) An ordinary geometrical rider stated in its easiest form, *i.e.* with the use of letters.

" If the side BC of a triangle be bisected in D and if the angles ABD, ADC, be bisected by the lines DE, DF, meeting AB, AC, in E and F, show that EF is parallel to BC."

Here we have a typical statement not at all easy to follow if it is merely read (reading aloud is peculiarly useless !) but quite easy to follow if the figure is drawn step by step. It is, therefore, again an easy exercise in reading, but it is essential that the boy should read and draw for himself, else much of the value is lost.

(3) Again an ordinary rider but expressed wholly in words and therefore requiring more intelligence in the reading.

" If two circles touch, show that any straight line through the point of contact divides the circles into segments which contain equal angles."

These may seem, as indeed they are, very easy exercises in reading ; but the very fact that boys misread them and draw quite wrong figures shows that they are effective exercises. Their simplicity and the immediate test they afford of accuracy in reading constitute their merit. Whether, having read them and translated them into figures, a boy can proceed to solve them or not is another question with which we are not now concerned ; our point is that the figure affords an immediate test of the character of the reading and that adequate reading of things of this sort is a considerable intellectual exercise.

In taking examples from Arithmetic we will avoid those which are geometrical in character as being more or less allied in principle with those already given.

(1) "If 5 tons of coal were put in the cellar 100 days ago, and since then 4 stones have been burnt per day, how much coal is there still in the cellar ? "

Not much it may be thought for a boy to go wrong over here, but experience shows that such an anticipation is wrong, and that plenty of boys will fail to read the question properly—their minds will not combine the second sentence with the first.

If a Master is taking such questions orally with a class, the boys having their books with the questions open before them, he will say " Look at question x and tell me what is the first thing to do (or to notice) " He will hope to get the answer " 400 stones have been burnt already." If so, it is a proof of good reading.

(2) A harder question : " In a certain district rates are 7s. 6d. for each £1 of rent. A man living there can afford £55 a year for rent and rates. What rent can he afford to pay ? "

Here a boy has got to translate the question into this form : for every

27s. 6d. he pays in all, he pays £1 in rent.   If he can do this, the rest is easy, but it is just in this preliminary step that he is most likely to fail, and he fails largely for want of intelligent reading.

(3) Not all arithmetic questions are of this brief form ; they may spread over several lines of print and it becomes necessary to read very carefully and pick out facts in an order quite different from that in which they are stated, *e.g.*

" The capital of a company consists of 15,000 shares of £1 each.   The business is afterwards enlarged by the issue of 2,000 shares, each of the nominal value of £1, for each of which 37s. is paid to the company.   If the rate of profit upon the capital employed remains the same as before, and if the new shareholders receive $5\frac{1}{2}\%$ for their investment, find what was the rate of interest on the shares of the company before its enlargement."

Here the reader has to ask himself some subsidiary questions : does the ' capital employed ' mean the nominal capital of £17,000 or the capital the company actually receives, viz. £18,700 ?   Does the ' $5\frac{1}{2}\%$ for their investment ' mean $5\frac{1}{2}\%$ on the nominal £2,000 or on the £3,700 paid for it ?   Not that the answers to these questions are doubtful, else the question would be bad because ambiguously worded ; but quite probably a boy will be in doubt at first as to the exact meaning, and he must get quite clear on these essential points.   But this is an important feature in all reading, hence the exercise is a good one just from this point of view.   .

Algebra more than either geometry or arithmetic is concerned largely with symbols, which constitute, it is true, a language of their own, but this part of it need hardly be considered in the present connection : we shall return to it when we come to consider expression ; meanwhile we need notice only one obvious point : algebraic expressions need careful reading, with a keen eye for minute differences, and close attention to arrangement and to brackets ; $2x$ must not be confused with x^2, nor $-x$ with x^{-1}, nor $\dfrac{1}{x+a}$ with $\dfrac{1}{x}+a$; nor must $\sqrt{a+b}$ be read as $\sqrt{a}+\sqrt{b}$, and it is essential to distinguish between $a+bc$ and $(a+b)c$ and so on ; while there is more than a mere analogy between the use of brackets and punctuation.

In the present connection, however, we are concerned with that part of Algebra which is trivial in the eyes of a mathematician, but which bulks large in the earlier stages of school work, " problems," as we call them, " leading to equations " ; the German name " Gekleidete Gleichungen " is more expressive.

(1) " A man bought 5 sheep and 8 calves for £30 ; he sold 4 of the sheep at a gain of 10s. each and 6 of the calves at a gain of £1 each, for £31.   What did he pay for each ? "

Here the test of intelligent reading is first to choose the right point of attack ; then having made the choice, to stick to it and fill in the details accordingly.   There are two obvious methods, the one suggested by the final question : let x£ be the cost of a sheep and y£ of a calf.   Then the boy has to appreciate the difference between cost per head and total cost and to form his two equations ; pretty certainly the weaker boys will fail over the second equation, really for want of grasp of the meaning of cost per head.   The alternative method (less likely to be adopted) is to notice that the profit on 4 sheep and 6 calves was altogether £8 ; hence they cost £23 ; the two equations follow as before but the second more simply.

We shall notice later on that problems of this kind, *after they have been solved*, provide excellent material for real though simple composition.

(2) " A broker sells a number of railway shares for £3,240.   A few days later, the price having fallen £9 a share, he buys, for the same sum, five more shares than he sold.   Find the number of shares transferred on each day, and the final value of his holding."

Quite straightforward, of course, but the second sentence is close packed, containing three separate details one or more of which would escape the careless reader, beside one which looks irrelevant but is not.   If any one is in doubt as to the reality of the test, let him try reading such a statement once and then attempt to reproduce it !

Examples might, of course, be multiplied indefinitely and of all degrees of difficulty : but probably this is enough to prove our thesis that elementary school mathematics, even in its narrowest aspect, quite apart from bookwork, inevitably affords excellent training in close reading, and that the reading is effectually tested, not by mere reproduction but by execution.

In lower and middle school mathematics little use is now made of books except as storehouses of examples.   No young schoolboy learns the elements of arithmetic or algebra from a book and even the bookwork of geometry is now mainly taught at the blackboard.   In former days, when boys were expected to learn Euclid from a book, they were faced at the outset with difficulties much more serious than those of the examples we have been considering—with the natural result that many of them failed altogether, or fell back on mere learning by rote.   This was inevitable, for the proper expression of bookwork involves considerable complexity of statement.   This must be faced and mastered at the proper time or real progress becomes impossible ; but the proper time is not at the outset, nor is the finished and classical form proper for a first presentation.   The true method of arriving at it we will consider later when we deal with expression.   Meanwhile an example or two of the sort of difficulty will serve as a reminder that the function of mathematics as a contributor to the power of reading is not soon exhausted.

Euclid's definition of a right angle is as follows : " If one straight line standing on another straight line makes the adjacent angles equal, each of these angles is a right angle."   And his thirteenth proposition is " If one straight line stands on another straight line, the adjacent angles are together equal to two right angles."

These two totally different statements were hopelessly confused with one another by the duller boys ; they both meant the same, *i.e.* neither meant anything.   If the master asked for one, he was as likely as not to get the other. Any form of complex sentence is difficult to some people—to the very young naturally, and to certain minds permanently.   No doubt total lack of interest accounted for a good deal, but the inability to grasp the relation of a conditional sentence—and especially of a doubly conditional sentence " *If* a straight line *standing . . .* " to the whole, seems sometimes to be real.

Even worse was the endless confusion between the propositions on parallels (direct and converse) and the parallels axiom.   " If a straight line falling across two other straight lines makes an exterior angle equal to the interior and opposite angle on the same side, *or* the two interior angles on the same side together equal to two right angles, the two straight lines are parallel."   " If a straight line falling across two other straight lines makes the two interior angles on the same side of it together less than two right angles, the two straight lines will meet if produced on that side of the cutting line."   " If a straight line falls across two parallel straight lines, it makes . . . the exterior angle equal to the interior and opposite angle on the same side, *and* the two interior angles on the same side together equal to two angles."

It wants a fairly strong head to find its way through any one of these long sentences and to keep them distinct, though they sound in general so much alike.

The enunciations of Euclid's second book supply further examples of sentences absolutely and beautifully clear when once they are mastered, but distinctly forbidding to the stumbling beginner.   In the text they necessarily stand at the head of the proposition and form its first presentment.   For the most part in sound teaching they come last ; the proposition is expounded

more simply, demonstrated or discovered, then formulated first in a simpler or cruder form, but finally (now the mind is ready to receive it) the classical form must be reached, and then quite properly learnt by heart.

These illustrations will serve as a reminder that as the subject progresses there is plenty of opportunity for exercising the power of close reading ; but as the growth of this power depends greatly on the exercise of the complementary power—expression, it will be better to turn to the opportunities offered in mathematics on this side.

This is the more important in that, as already pointed out, practice in close reading cannot be avoided, while unfortunately training in expression can be shirked and too often is. Not only so, but some measure of success, often enough to satisfy minimum examination requirements, can be secured in spite of such neglect. None the less it is utter treason, both to mathematics and to education generally, and the success is delusive.

The matter has two complementary aspects :

(1) Mathematics offers endless opportunities for training in exact expression ; and

(2) Without careful attention to expression complete clearness of thought cannot be attained.

The latter principle concerns the teacher of mathematics more immediately, so we will consider it first.

It is a commonplace of teaching experience that one never knows a subject properly until one has to teach it. This means that even when knowledge is fairly full and complete, obscurities are apt to exist which are not detected till one tries to expound the subject to someone else. If a teacher does his work properly, he finds himself driven to think over his subject matter afresh each time he deals with it, he learns from his own mistakes and defects in exposition, and by degrees attains to full mastery both of material and expression.

But look at the matter from the other end, that of the boy. It is common enough for a boy to say " I see that it is so, but can't explain it " ; a fuller and more exact description of his state of mind would often be " I see *that* it is so, but I don't see *why*," and this means that his vision or understanding is partial and hazy. Let us take some detailed illustrations of this.

If a class is asked " If the diagonals of a parallelogram are equal, what particular kind of parallelogram is it ? " they will reply without much difficulty " A rectangle " : but if they are asked why, the answer is not so easy. For to give the answer is to give what geometricians call a proof, *i.e.* to put the obvious fact into relation with other known facts, and so to make clear its reason and true nature.

Much of the earliest work in geometry is just of this character ; it is sometimes contemptuously called " proving the obvious," but it is none the less an important contribution towards securing both fulness of knowledge and clarity of thought. Now if the fact and its proof or explanation are taught dogmatically the main virtue of the process is lost ; the proper way, now happily generally adopted, is to make boys try to find the proof themselves ; their efforts will be stumbling and clumsy ; they will see dimly at first, will fasten on all sorts of irrelevant matter and make actual mistakes ; they can be guided to a certain extent : " look for a pair of triangles that may be congruent " ; " are the triangles you have chosen of any use ? " : " have you got a proper set of equal elements in your triangles ? " and so on ; but subject to such guidance, they must learn to see for themselves and they have not seen properly until they can give expression to what they see, in the first instance perhaps merely by making significant marks on their figure, but finally by putting it into words, which again will need much criticism and polishing before the whole is in satisfactory shape.

Take another example, the proof that an external angle of a cyclic quad-

rilateral is equal to the opposite interior angle. The proof will probably first be evolved in some such form as this :

The angles x and a are together equal to two right angles ; also y and a are together equal to two right angles ; therefore $x + a = y + a$, therefore $x = y$.

This is right and clear, but becomes more effective, clearer and more forcible when it is put into a single sentence : the angles x and y are equal because each is the supplement of a. Just because the expression is briefer it is easier to grasp as a whole, and the point is better brought out.

Often a single word becomes of value : a boy will say " The angles at the circumference are equal, because they are half the angle at the centre." Here one wants to have a word supplied, which should be emphatic, and which clinches both the expression and the vision of the proof : " because they are all half the *same* angle at the centre " ; and until a boy can supply this word he does not see the point exactly.

As an example of a slightly different kind take the exercise of formulating the enunciation of a proposition, the truth of which has been perceived in a general sort of way from a figure : having drawn a set of parallel lines on the board and a pair of transversals the teacher proves or gets a proof that if the intercepts on one transversal are equal, so are those on the other ; but if at the end he asks for the enunciation, he will get all sorts of errors and imperfections ; but if he handles the matter rightly will ultimately get the class to state it clearly—the essential thing being to begin at the right point. " If a set of parallel lines make equal intercepts on one transversal, they do so also on any other."

Arithmetic affords plenty of illustrations of the same principle that clearness of thought is achieved only by constant effort after clearness of expression.

A child asked " if 2 balls cost 6d. what did one cost ? " will answer easily enough " Threepence " ; but if the question is " if $2\frac{1}{2}$ yards of cloth cost 15s. 9d. what was the price per yard ? " he may be quite in a fog as to whether he should add, subtract, multiply or divide ; and if he perceives that it is a division sum he may still fail to distinguish divisor from dividend. He must be got to see, consciously and explicitly, first that it is a division sum, and second that 15s. 9d. has to be divided by $2\frac{1}{2}$.

Here and in many such cases it becomes important to stress a particular word ; a boy will say divide 15 *and* 2 ; it is perhaps better to check the use of divide 2 *into* 15 and insist on the uniform use of *by*.

In these days the use of the elementary method " reduction to unity " seems to have become unpopular ; but it has this great merit, that it compels (or would compel if the teacher were not negligent) a clear statement and sometimes a real little piece of rearrangement, *i.e.* of composition ; and without this it is as unsatisfactory a tool as the method of proportion unintelligently used.

The simplification of an elaborate fraction gives an example of a different kind : here words are unnecessary, but clearness of arrangement is indispensable to success ; it cannot possibly be achieved without clearness of thought, and the effort to attain it involves the effort to see clearly. It is surely a good exercise in " composition," although the sentences are expressed in symbols, not words.

The translation of simple algebraic expressions into language is at once useful training in expression and perhaps the surest way of mastering one of the essentials of algebra. For instance $a - (b + c)$ should be translated : " Add c to b and subtract the result from a " ; or this can be extended : " We have two little sums to do, an addition and a subtraction, and the addition must be done first." Instead of further multiplying general examples of this kind it may be more useful to give some actual class-room experiences.

The following then are examples of " English as she is spoke " recorded with approximate accuracy. Absolute accuracy is difficult, for what boys say

is often so incoherent that it is almost impossible to remember exactly; their words "like the baseless fabric of a vision, leave not a rack behind." Unless they are jotted down instantly, they generally go down in an already improved form. Still what follows should be a sufficient indication of what happens, and teachers and observers have only to keep their ears open to collect at first hand plenty of instances for themselves. It will be understood that we are dealing at present not with "Essays," nor even paragraphs, but merely with isolated sentences, verbal answers to questions.

(1) Master: "What is the first thing to do with this equation?" Boy: "L.C.M."

(2) Master refers the class to question in their books open before them; the actual question was: "Why is $\sqrt{49^3} = 7^3$?" Boy: "Seven times seven."

(3) Master, referring to a mensuration question done last lesson, the figure of which he has just redrawn on the board, "What was the first thing we did last time?" Boy: "By joining the roof."

Here we have there typical answers, all of which (even the last) showed that the boy had some glimmering of the truth in his mind, but failed to express it. The first easily lends itself to treatment which will expose the ignorance of some, possibly of the original contributor, and give others the chance of criticising one another's attempts. Naturally the teacher asks "What about the L.C.M.?" or "What is the use of the L.C.M.?" and by degrees gets the proper reply "Multiply both sides by the L.C.M. of the denominators." In this particular case, one of regular routine, the answer ought to have come at once clear and exact; that it did not showed that the work was not well known, and pretty certainly that the particular boy's notions were hazy. The second question is a much more difficult one, or would have been if the boys had not just been dealing with simpler ones of the same kind; but it illustrates very well the gulf that often yawns between dim perception resulting in partial or obscure expression and clear perception leading to clear expression.

(4) "How do you square ab^2?" "Multiply *them* together."

This is a simpler, but very common type and one has constantly to ask "What is 'it'?"—or as here 'them'?

(5) "*Which side* of the triangle has to be produced?" Boy: "Out here." Obviously he did not attend to the form of the question.

As might be expected questions requiring a complex sentence for answer are sad stumbling blocks.

(6) "How do you test the accuracy of your solution?" (of an equation). Boy: "Substitute 3 for x."

This again indicates that the boy has heard of testing his answers, but quite possibly and not improbably he is still in the dark; so the further question is essential, "Well, what then?" The complete answer is "Substitute 3 for x in the two sides of the equation separately; if the results come the same the solution is right." It is quite a serious exercise in composition to many boys to produce this or any tolerable equivalent, even if they understand and appreciate the process.

(7) An answer that required only a moderate amount of improvement: "The ratio of two segments on the one side is equal to the ratio of two segments on the other side." Here a further question or questions are wanted to get the completion "equal to the ratio of *the two corresponding* segments on the other side."

(8) An attempt at formulating the statement of a fact already perceived in a figure: "An arc of a circle joined to any other point of the circle is equal to" (here the record broke down). What the boy was trying to say was that two angles are equal, or one half the other. He began at the wrong point and a good deal of subsequent questioning was necessary before the class evolved (as they did ultimately) a tolerable form of words.

Sometimes, indeed generally, the collective efforts of a class towards improving a poor but not hopeless answer are very interesting. Here is a specimen

from a Fifth Form lesson on Mechanics. Having drawn this figure on the
board the Master asked : " Where does P go ? "   First boy : " Produce the
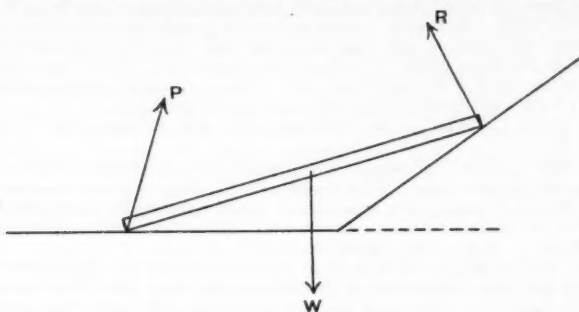
FIG. 1.

weight, and where it meets R, then join that point to the end of the plank."
Second : " Where the line of action of W meets the normal reaction of the
plane *that* will be a point through which the reaction of the ground goes."
Third : Omit the ' that.'
Fourth : " The join of the intersection of the lines of action of the weight
and the normal resistance of the plane with the base of the plank will be the
line of action of the reaction."
(9) Master : " What do you mean by the square root of a number ? "
Here the early answers were very weak, not unnaturally, for the question is
not an easy one to young boys.   One perhaps worth quoting was " The square
root is a number multiplied by itself."   However they ultimately evolved this
weird statement : " The square root of a number is a factor when multiplied
by itself produces the number."   No one seemed to feel this unsatisfactory
so it was written on the board ; then, but rather slowly, quite a few boys
began to notice the absence of the essential " which."
(10) The Master had drawn this figure on the board and the question was
asked " What has he done ? "

FIG. 2.

First Boy : " Drop a perpendicular on to the parallel lines."
Second—mere hopeless confusion.
Fourth and Fifth : " He's erected a perpendicular on AB touching the top
line at P."
Sixth : " Take the mid point of AB and erect a perpendicular on AB and
call the point where it cuts one of the parallel lines P."

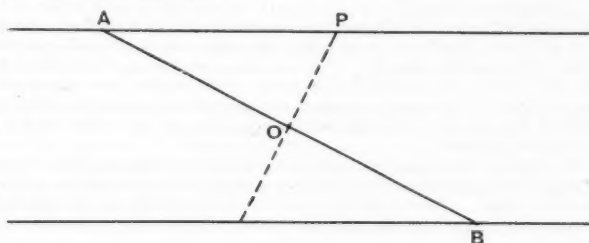(11) After drawing some graphs a boy wrote as his conclusion " $2x$ gives $4x - x^2$ its greatest value which equals 4." After some questions meant to help him to see more clearly he was left alone and produced the correct statement : " $4x - x^2$ has its greatest value when x is 2." He was still in the dark as to the error in his original form, and it took more questions and further consideration on his part before he arrived at the correction : " x equals 2 gives $4x - x^2$ its greatest value."

As usual this was not merely a matter of English ; there was some confusion in his mind as to the meaning of the graph itself which was not fully cleared up till he got the expression right. But it looked as if the second form with its subordinate noun sentence was inherently more difficult to him than the first where the subordinate sentence was adverbial.

It is unnecessary to multiply examples at length, but there are some isolated gems (again quite typical) taken from boys' (and teachers' !) actual speech.

" Walk at right angles to the point C."

" $OA = OB$ because it is radii " ; " line OM is in the middle of AB " ; " because they are the same radii " (for radii of the same circle) ; " The perimeter simply is finding from one point to another."

" A group of 2 taken half times."

" The triangle makes it 180°."

" The centre of the circle is the perpendicular bisector of three of them."

Poor speech is naturally followed by bad writing. A class had been set to write brief answers to some revision questions : here are three specimen answers from three different books—all ticked off by the Master as right :

" From two equal distances along the lines of the bisected angle, draw an arc from each. Where this crosses draw line from angle cutting the cross ; the angle is therefore bisected."

" Take the two sides of the rectangle and draw them in one line ABC and then describe a semi-circle on the line ABC. At the point B produce a line upwards to meet the semi-circle at D, this line BD being the side of the square."

" To find the $\sqrt{7}$ two numbers whose product amounts to 7 must be taken and two lines end to end must be made in that proportion. The mid point is then found (trial or calculation) and a semi-circle drawn on it. Produce the point where the two lines meet to the line till it reaches the circumference and the length of the line will be $\sqrt{7}$."

These were quite typical of the general style of answer. It will enforce the point to give the titles of ' essays ' set to the same form in the same term : Health, Wealth, The Ocean, Dress as an indication of character. A rolling Stone ... The pen is ... Suitability of dress to action—and to a lower Form reported particularly dull " ... in 2023."

All teachers probably will recognise the picture : examples of the sort are to be heard in very many lessons and seen in many exercise books. Unfortunately teachers too rarely recognise the opportunities presented to them ; they are anxious to " get on " and will perhaps say " Yes, I think I know what you mean, that will do." On the other hand, sometimes it is to be feared that they will find fault with the boy, possibly even bully him, quite failing to draw the distinction between " howlers " that should be treated severely, and crude attempts at self-expression which need encouragement.

It is comparatively easy to learn a correct statement by heart and to reproduce it : but that is not to learn to speak correctly. This is a vastly harder thing, and can be attained only by making attempts which will at first be very imperfect. If the attempt is not made, or when made, is discouraged, no sound progress is possible any more than it is if the feeble attempt is allowed to pass without the effort after improvement.

So far we have been concerned with answers and statements requiring only a single sentence, simple or complex. Let us see what opportunities quite

elementary mathematics afford for composition on a larger scale. Here is a commonplace " problem " in algebra :

" A and B go for a holiday together, A starting with £15 more than B. At the end of a week A has spent ¼ of his money and B ⅓ of his, and together they have £58 left. Find what each started with."

In any such question a good teacher recognises that when a boy has got an answer he has not finished ; it is very important, for reasons that need not be enumerated here, that he should satisfy himself that the answer is right. This involves, not going through his algebra again, but going through the whole story with the data supplied by the answer and seeing if it works out properly. This is quite an excellent exercise in composition in the proper sense, *i.e.* the putting together of a series of ideas in good order, omitting nothing essential but excluding all that is irrelevant.

In the particular case quoted, several boys were asked to make the verification orally : there was some mutual criticism ; finally as an experiment the whole class was asked to write it out.

As might be expected after this preparation it was decently done, but there was still quite enough imperfection and variation to show that it was well worth doing, and that it was a very real exercise in composition.

Here are some typical attempts ; the answers worked with were those obtained by some boy and chanced to be wrong, but that does not affect the issue.

1. " A started on a holiday with £60 and B started with £45. At the end of the first week the former had spent ¼ of his, leaving £45, and B had spent ⅓ of his, leaving £30. They had at the end of the week the sum of £75 altogether."

2. " A started with £60, therefore B started with £45. At the end of the week A had spent £15, and B £15. Therefore A had 45 left and B £30 left. Therefore together they had £75 left."

3. " A starts with £60, B starts with £45. At the end of the week A spent a quarter of this money, that equals £45 he had left, and B at the end of the week spent ⅓ of his money, that equals £30 he had left."

4. " A and B went for a holiday. A had £60 and B had £45. At the end of a week A had left £45 and B £13 left. They had £105 when they started together. Therefore A started with £67 10s. and B with £37 10s."

Minor eccentricities were to be found, confusion of tenses and the like ; one boy put in irrelevant remarks.

This was, so to say, prepared work, as they had been through it in class first ; so to carry on the experiment they were asked to write the story of the next question unaided. Naturally there was much more variation and much omission of essential points. There were only two or three reasonably complete accounts among the 25. One of the good versions (though spoiled by the barbarous use of =) ran : " A tradesman buys 30 bicycles at £13 each = £390. He then sells ⅔ of them (20) at £18 5s. = £365, 3 more at £17 15s. each = £53 5s., and the remainder at £15 = £15 × 7 = £105. Therefore profit he gains = £365 + £53 5s. + £105 = £523 5s. − £390 = £133 5s."

A second made it easier to himself by omitting the prices while retaining the totals. The majority merely copied the prices, omitting the essential totals, and not infrequently copied the profit from the book (which was different), showing that they had failed to grasp the purpose of the exercise as a whole. For instance : " A tradesman buys 30 bicycles at £13 each, he sells 20 or ⅔ of them at £18 5s. each, 3 at £17 15s. each, and the end of the season sells the remainder at £15 each. When he has sold all the bicycles he made a profit of £70 15s."

This example given at length shows what excellent material for composition of a simple, but exacting, type is afforded by this class of work—the ordinary problems in algebra and arithmetic dealt with in the lower Forms.

As boys progress in algebra and geometry there is plenty of room for exercise

in composition of a severer kind—very obviously in riders, but also in the book-work of both subjects ; in mechanics and the higher branches of school and university mathematics this is still more the case. Of course, if the bookwork is merely learnt and reproduced, much of the advantage is lost, but this is a poor way of doing the work, even for boys. While, if it is permissible to say a word to teachers themselves, they at least have a great opportunity, going over the same ground again and again, year after year, of improving their exposi-tion, not simply by such variation of material and illustrations as the subject permits, but also by clarifying, condensing and polishing the form.

To sum up, it is suggested that in essence Mathematics is the easiest of all subjects. It deals primarily with the simplest of all mental operations, counting, and the simplest cases of space connections. It draws the minimum of material from the outside world, disregards the endless variations of cir-cumstances, form and character. When it deals with physical facts it reduces them to their barest outline ; personality and the endless complications of human nature it ignores entirely.

Just because its range is in a sense so narrow it achieves a certainty and accuracy such as no other subject admits ; it is thought at its purest, absolutely certain of its results, and absolutely clear in their expression.

While, therefore, it is of all subjects the easiest in essence, it is also the most exacting ; ignorance and obscurity of thought cannot here be covered by a cloud of words, however beautiful.

If this essential characteristic of the subject is ignored, it is difficult to see what claim it has to be regarded as a necessary element in the education of every individual. On the other hand it is equally certain that without per-sistent striving after clearness of expression no one will learn any mathematics worthy of the name.

Hence we have clearly the double conclusion : first, that mathematics has a quite special place in the general training in the use of speech ; and second, that if a teacher of mathematics neglects this side of his duty, he is failing to teach the subject itself, even more emphatically than the teacher of any other subject who should be guilty of the same neglect. W. C. FLETCHER.

GLEANINGS FAR AND NEAR.

221. Had only the name of Sir Isaac Newton been subjoined to the design upon his monument, instead of a long list of his discoveries, which no philo-sopher can want, and which none but a philosopher can understand, those, by whose direction it was raised, had done more honour both to him and to themselves . . .

Next in dignity to the bare name . . . would be this epitaph : " Isaacus Newtonus, naturae legibus investigatis, hic quiescit."—Murphy's edition of the works of Dr. Johnson. *Essay on Epitaphs*, ii. 272-3.

222. The sea running pretty high . . . our hero . . . began to be squeamish . . . while the governor, . . . experienced in these disasters, slipped into bed, where he lay at his ease, amusing himself with a treatise on the Cycloid, with algebraical demonstrations, which never failed to engage his imagination in the most agreeable manner. . . .

At the mention of " dead lights," the meaning of which he did not under-stand, the poor governor's heart died within him. . . . He fell on his knees in the bed, and fixing his eyes on the book which he had in his hand, began to pronounce aloud with great fervour : " The time of a compleat oscillation in the cycloid, is to the time in which a body would fall through the axis of the cycloid *DV*, as the circumference of a circle to its diameter . . ."—Smollett, *Peregrine Pickle*, ii. c. 1.

EUCLID AND HIS SUCCESSORS: SOME CONFUSION AND A WAY OUT.

BY G. GOODWILL, B.Sc.

MANY of us who learned Euclid in our schooldays, feel that something substantial has been lost in the training in precise expression and logical statement that it gave. Even those mathematical teachers who would admit no such thing, must admit, if their experience is anything like my own, that many of their colleagues, particularly science teachers, have deplored the loss of what was at least a standard of reference for logical form.

Our answer to them has been that Euclid's formulation of Geometry is quite unsuited for the purpose of teaching the subject, and to most of us this has been a sufficient justification for letting Euclid go. As mathematical teachers, our purpose has been to teach Geometry, and our schemes have been arranged to carry this into effect. Being convinced as we are that spatial experience must be made the basis of our geometrical teaching, and that consequently the inductive method must be used, we have not questioned our rightness in abandoning Euclid's deductive treatment; and thus encouraged, the lament of our colleagues and our own misgivings have been lost sight of.

My own feeling is that in this way the issue has been confused. I think that the result of misgivings in the back of our minds has been to make our schemes more formal than they should be, to be really effective. I am very doubtful whether if a really substantial and coherent body of geometrical knowledge was covered in the school course, this could be reduced to anything like a rigid sequence in the last stage—indeed the report admits that we have no experience of this having been done. The inference is, I submit, that the method in this subject should be frankly inductive, as it is in every other science that deals with experience, and as to its final reduction in the last stage, I personally do not admit that the most rational view of an ordered structure presents it as a linear sequence.

But on the other hand, need we abandon Euclid? Though unsuited for the teaching of Geometry, could not the study of Euclid as a classic, as a play of Shakespeare is studied, serve as an unrivalled means of training the power of precise expression and logical statement, and as a standard of reference in this respect, which would be valuable in connection with all branches of the curriculum? If a lesson once a week, or even less frequently, could be devoted to Euclid, including very easy riders, I think there are few who would not admit that the time was well spent, even though quite a limited amount of the text was covered.
G. GOODWILL.

223. "Every deep enquirer," said Barrow to Newton, "has discovered more than he thought prudent to avow: almost every shallow one throws out one more than he has well discovered. Remember how Galileo was persecuted for his discoveries." To Newton's remark that it was because Galileo lived under a popish government, Barrow retorted: "My friend! my friend! all the most eminently scientific, all the most eminently brave and daring in the exercise of their intellects, live and have ever lived, under a popish government. There are popes in all creeds, all countries, in all ages." The conversation may be "imaginary," but when Landor wrote it perhaps he spoke more truly than he knew. The interpretation given to the facts of Nature which we associate with the name of Darwin was challenged from the moment of its promulgation.

224. I found (poor Lady Charlotte) doing problems in Euclid instead of flirting with Lord Ilchester.—*Harriet Countess Granville* (to the Duke of Devonshire), Sept. 1817.

CONSTRUCTIONS IN GEOMETRY. WHAT IS LEGITIMATE?

By Prof. C. Godfrey, M.V.O.

Construction. As we depart widely from Euclid's ideas in the proofs of Stage B, it seems to be thought by many people that go-as-you-please methods are good enough for constructions as well. Now, "trial" methods of construction are sometimes useful in more advanced work; but they are tolerated only when a geometrical method is impossible; they are means to get a solution, but there is no instruction to be had from such a method. We would therefore urge that a certain severity be still maintained in problems of construction.

Euclid's instruments were an ungraduated ruler and a compass that might not be used to transfer a length, this restriction being overcome by Prop. I. 2. We have added graduations to the ruler, a protractor and a set-square. These means of measuring lengths and angles are useful in enabling us to set an identical problem to a class; the set-square gives us a method of drawing parallels and perpendiculars, which is a legitimate alternative to Euclid's ruler and compass method; for extreme accuracy the latter method is preferable for the perpendicular, as the right-angle of the set-square may be false.

We have thus enlarged our list of legitimate operations, but liberty must not degenerate into licence; to make the game worth the playing there must be rules, even in Stage B.

It will be useful to cite two examples of licence noticed in the answers in a recent examination. In one case it was necessary to find a point P on a certain line AB such that another line XY should subtend a right-angle at P. Many candidates arranged their set-squares with the right-angle on AB, and the arms of the right-angle passing through X and Y. This showed resource, but was not playing the game.

In another case it was required to draw a figure of a wheel of a given radius, with a brick of given thickness acting as a block to prevent the wheel from running down hill.

The construction favoured by candidates was to set the compass to the thickness of the brick, and to fit in this length as best they could between wheel and ground. The only right way is to draw a parallel to the ground at the right distance and to let it cut the circle.

It will appear arbitrary to admit one method and to exclude another, unless the rules of the game are formulated. What are they?

Constructions eventually reduce to the determination of a point or a line. *A point must be determined by the intersection of two lines;* and this may generally be envisaged as the intersection of two loci. It is also lawful to mark a point at a given distance and bearing from a given point.

A line may be determined in the following ways:—(1) by joining two points; (2) by drawing a line through a point parallel or perpendicular to a given line; (3) by drawing directly a tangent to a circle from a point, or a common tangent to two circles.

For instance, it is not legitimate to draw directly a line through an external point to make say 70° with a given line, though it is possible to make a fairly good job of this with a protractor.

There are some who would exclude method (3) above. But it may be argued that this is the practical draughtsman's method; and that to draw a common tangent to two circles is a proceeding not less accurate than to draw a line through two points, which indeed are generally small circles.

Another point to be mentioned concerns the description of a construction. In the wheel and brick problem no description was asked for and most can-

didates gave none : the closest scrutiny was necessary to decide whether they had drawn the parallel before or after finding the point on the circumference ; the examiner, not the candidate, had the benefit of the doubt. Boys should be trained to exercise judgment in describing their constructions. No one wants a long account of how each angle is bisected and each perpendicular is drawn ; details of this sort may be taken for granted ; but in each problem of construction there is one point of interest ; let the boy decide about this point, and make clear in his description how he has dealt with it.

<div align="right">C. Godfrey.</div>

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225. I visited the most famous Newton, who received me with great kindness. if only for Volder's sake, as whose pupil I introduced myself. We spoke chiefly of the system of the universe.

First. The motions of the planets exactly observe the laws of gravity, which amount in the main to these. 1. Gravity always acts with the same force on the same portion of matter, excepting the resistance, which arises from the surface of the body moved. So that bodies of equal bulks would fall with equal velocity *in vacuo*—*i.e.* if there were nothing sensibly to retard their motion. 2. The velocity with which heavy bodies fall, is in a duplicate ratio to the spaces traversed. But as these proportions suit any one of the sections of a cone, if a heavy body, instead of falling, be conceived as whirling round by the force of gravity, it will describe one of the conic sections. And since these laws hold in all planets, we must imagine the ethereal matter to be so rare, as not sensibly to interfere with these laws of motion. But the illustrious author admits this rarefaction to be so great, that between each part of the ethereal matter spaces are interposed void of all matter.

I objected then that motion is not uniform, but changes at each of these spaces ; because bodies passing from one medium into another are liable to refraction. But these media are of all most different, the one being entirely void, the other close packed with matter. The answer I did not well understand.

A second difficulty is derived from comets. For they move with equal velocity whether carried away with the stream of the vortex, or making head against it. For sometimes they fall far below Jupiter's orbit. The main endeavour of astronomers at this day is to determine the orbits of comets from given observations. But a somewhat longer time is needed for this purpose, the first trustworthy observations having been made only by Tycho.

I found him most ready to do me a service ; he declared he would most willingly have introduced me to the Royal Society, but as he was engrossed at that hour with Mint business, it could be most conveniently done by Dr. Halley. So he immediately writes a letter of introduction to him:

" The gentleman who brings you this is one of the chaplains to the Dutch ambassadors. I beg the favour that by the leave of the R. Society you would introduce him to see one of their meetings. He has heard Monsr. Volder's lectures and has a curiosity about Mathematical and Philosophical things. If he brings a friend with him, I beg the favour that you will treat them with respect. I'm your humble servant, Is. Newton.

Jerom-Street, June 2, 1702.   [F. Burman's *Journal*, June 13, 1702.]

226. *Mathematical Strictures.* Part I.—*A book of arithmetical examples, without answers.* A discussion in the appendix about the equation of payments seems to have suggested to the author to call the book a book of *Strictures*—the consequence of which will be, that those who want examples will not think of it, and those who buy it from the title will be disappointed. —*Athenaeum,* 1849.

BLAISE PASCAL.

By the Rev. J. J. Milne, M.A.

(b. 1623, d. 1662.)

Most mathematical students are familiar with Pascal's statement, made in a communication to a learned society in Paris in 1653, that he had a number of mathematical treatises, chiefly geometrical, ready for publication, amongst them being the following: *Conicorum opus completum, et conica Apollonii et alia innumera unicâ fere propositione amplectens ; quod quidem nondum sex decimum aetatis annum assecutus excogitavi ; et deinde in ordinem congessi.*

As reference is often made to his *Essais pour les coniques* written in 1640 when he was seventeen years old, we give a translation for the benefit of those of our readers who are not acquainted with it.

There is in the library of the M.A. a copy of Pascal's works in five vols. (1819), of which vol. iv. contains the *Essais pour les coniques.*

SUGGESTIONS FOR A MORE GENERAL TREATMENT OF CONICS.

1. *Def.* 1. When several lines pass through the same point, or are all parallel to one another, they are said to be of the same order, or the same pencil or sheaf, and the collection of these lines is called an order of lines, or pencil, or sheaf.

Def. 2. By the term section of a cone we mean the circumference of a circle, ellipse, hyperbola, parabola and rectilineal angle ; according as a cone which is cut parallel to its base, or through its vertex, or in the three other ways which give the ellipse, hyperbola and parabola, gives in its plane either the circumference of a circle, or an angle, or the ellipse, hyperbola or parabola.

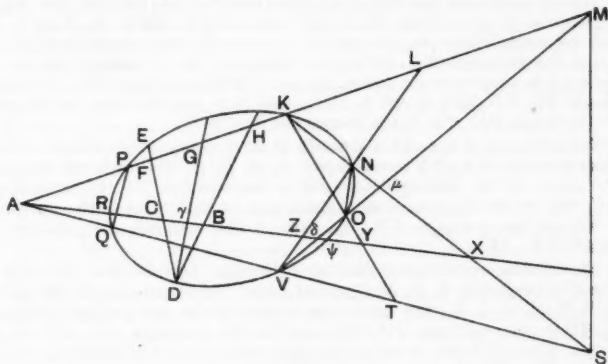*Def.* 3. By the term line, taken alone, we mean a straight line.

Fig. 1.

Lemma I.

2. If in the plane *MSQ* (Fig. 1) from the point *M* are drawn the two straight lines *MK, MV*, and from the point *S* the two straight lines *SK, SV*, and if *K* is

the intersection of MK, SK, V the intersection of MV, SV, A the intersection of MA, SA, μ the intersection of MV, SK, and if through two of these four points A, K, μ, V, which are not in the same straight line with the points M, S, say the points K, V, a circle be drawn cutting the lines MV, MP, SV, SK in the points O, P, Q, N; I say that the lines MS, NO, PQ are of the same order (*i.e.* are concurrent).

LEMMA II.

3. If through the same straight line there pass several planes which are cut by another plane, all the lines of the sections of these planes are of the same order with the straight line through which the aforesaid planes pass.

4. These two lemmas being granted, and certain easy deductions from them, we will prove that with the same data as in Lemma I., if through the points K, V (Fig. 1) be drawn any conic section which cuts the straight lines MK, MV, SK, SV in the points P, O, N, Q: the straight lines MS, NO, PQ will be concurrent.

This will be a third Lemma.

5. From these three lemmas and certain deductions from them we will obtain all the elements of conics, *i.e.* all the properties of diameters and latera recta, tangents, etc., the restitution of the cone from almost every kind of data, the description of the sections of the cone by points, etc.

In doing which, we state the properties which we are dealing with in a way that is more general than is usual. For example, take the following:

6. If in the plane MSQ (Fig. 1) in the section of the cone PKV the straight lines AK, AV are drawn cutting the curve in the points P, K, Q, V; and from two of these four points which are not in the same straight line with the point A, as the points K, V, and through two points N, O, taken on the curve, are drawn four straight lines KN, KO, VN, VO, cutting the straight lines AV, AP in the points L, M, T, S, I say that the ratio compounded of the ratios of PM to MA and of AS to SQ is the same as the ratio compounded of the ratios of PL to LA and of AT to TQ.

7. We will prove also that if there are three straight lines DE, DG, DH (Fig. 1) which are cut by the straight lines AP, AR in the points F, H, C, γ, B; and in the straight line DC, the point E is given, the ratio compounded of the ratios of the rectangle $EF \cdot FG$ to the rectangle $EC \cdot C\gamma$ and of $A\gamma$ to AG is the same as that compounded of the ratios of the rectangle $EF \cdot FH$ to the rectangle $EC \cdot CB$, and of AB to AH; and it is also the same as the ratio of the rectangle $FE \cdot FD$ to the rectangle $CE \cdot CD$.

8. Consequently, if through the points E, D there be drawn a conic cutting the straight lines AH, AB in the points P, K, R, ψ; the ratio compounded of the ratios of the rectangle $EF \cdot FG$ to the rectangle $EC \cdot C\gamma$, and of γA to AG, will be the same as that compounded of the ratios of the rectangle $FK \cdot FP$ to the rectangle $CR \cdot C\psi$, and of the rectangle $AR \cdot A\psi$ to the rectangle $AK \cdot AP$.

9. We will also show that if four straight lines AC, AF, EH, EL (Fig. 2) intersect in the points N, P, M, O, and if a conic cuts these lines in the points C, B, F, D, H, G, L, K; the ratio compounded of the ratios of the rectangle $MC \cdot MB$ to the rectangle $PF \cdot PD$ and of the rectangle $AD \cdot AF$ to the rectangle $AB \cdot AC$ is the same as the ratio compounded of the ratios of the rectangle $ML \cdot MK$ to the rectangle $PH \cdot PG$, and of the rectangle $EH \cdot EG$ to the rectangle $EK \cdot EL$.

10. We will also prove the following property, first discovered by M. Desargues, of Lyons, one of the great minds of this age, and one of those who are most skilled in mathematics, in conics as well as in other branches, whose writings, although few in number, have given ample testimony of his genius

to those who are interested in this subject. I wish to state that the little which I have discovered in conics I owe to his writings, and that I have tried to imitate, as far as I could, his method of dealing with this subject in which he has made no use of the triangle through the axis in a general treatment of all the sections of a cone. The wonderful property referred to is as follows :

If in the plane MSQ (Fig. 1) there is a conic PQV, in the curve of which four points K, N, O, V being taken, the straight lines KN, KO, VN, VO are drawn so that only two straight lines are drawn through any one of the four points, and if another straight line is drawn cutting both the conic in the points R, ψ, and the straight lines KN, KO, VN, VO in the points X, Y, Z, δ ; I say that as the rectangle $ZR . Z\psi$ is to the rectangle $YR . Y\psi$, so is the rectangle $\delta R . \delta\psi$ to the rectangle $XR . X\psi$.
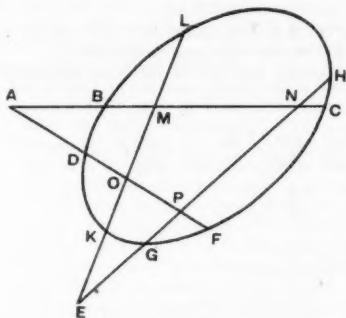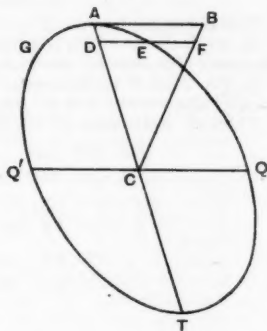

FIG. 2. FIG. 3.

11. We will also prove that if in the plane of the hyperbola or of the ellipse or of the circle $AGTE$ (Fig. 3), of which the centre is C, we draw the line AB touching the curve at the point A, and having drawn the diameter AT we take AB of such a length that its square is equal to one-fourth of the rectangle of the figure (i.e. one-fourth of the rectangle contained by the diameter and its parameter), and we draw CB, and also any straight line as DE, parallel to AB, cutting the curve in E, and the lines AC, CB in the points D, F ; if the section AGE is an ellipse or a circle, the sum of the squares on DE, DF will be equal to the square on AB ; and in the hyperbola the difference of the same squares on DE, DF will be equal to the square on AB.

12. We will also deduce some problems, for example :

From a given point to draw a straight line touching a given conic.

To find two conjugate diameters inclined at a given angle.

To find two diameters inclined at a given angle and in a given ratio.

13. We have several other problems and theorems, and various deductions from the above ; but the mistrust which I have of my own little knowledge and ability does not permit me to proceed any further with the work until it has been submitted to the examination of men who are qualified to form an opinion about it, who may induce me to take the trouble to go on with it ; and then, if they think that it is worth continuing, I will with God's help pursue the subject as far as power is given me to do so.

[Notes by the translator.

The property in Lemma I. is given for the line-pair by Pappus in Lemmas 12 and 13 on Euclid's porisms, where he proves Pascal's Theorem first when

the lines are parallel, and then for the general case. Pascal in *Def.* 2, recognising that the line-pair belonged to the family of the sections of a cone, probably inferred that the property was true for the circle (Lemma I.), and also for the ellipse, etc. (in Lemma III.).

In § 6 the conic pencil $K(PQON) =$ the c.p. $V(PQON)$;

$$\therefore \text{ the range } (AQTS) = \text{the range } (PAML);$$

$$\therefore \frac{AT}{AS} : \frac{QT}{QS} = \frac{PM}{PL} : \frac{AT}{TQ}, \text{ etc.}$$

In § 7, part 1, the range $(ACB\gamma) =$ the range $(AFHG)$;

$$\therefore \frac{AB}{A\gamma} : \frac{CB}{C\gamma} = \frac{AH}{AG} : \frac{FH}{FG}, \text{ etc.}$$

This is Pappus Lemma 3.

In § 7, part 2. This reduces to $FH \cdot BA \cdot CD = AH \cdot BC \cdot DF$, which is Menelaus' Theorem for the triangle AFC cut by the transversal HBD.

In § 8. Let O be the centre of the conic, Oa, Ob, Oc the semi-diameters parallel respectively to PK, $R\psi$, DE.

Then, by Apollonius, iii. 16-23,

$$\frac{EF \cdot FD}{FK \cdot FP} = \frac{Oc^2}{Oa^2}, \quad \frac{AR \cdot A\psi}{AK \cdot AP} = \frac{Ob^2}{Oa^2}, \quad \frac{CR \cdot C\psi}{CE \cdot CD} = \frac{Ob^2}{Oc^2};$$

$$\therefore \frac{EF \cdot FD}{CE \cdot CD} \times \frac{CR \cdot C\psi}{FK \cdot FP} = \frac{Ob^2}{Oa^2} = \frac{AR \cdot A\psi}{AK \cdot AP};$$

$$\therefore \frac{FK \cdot FP}{CR \cdot C\psi} \times \frac{AR \cdot A\psi}{AK \cdot AP} = \frac{EF \cdot FD}{CE \cdot CD}$$

$$= \frac{EF \cdot FG}{EC \cdot C\gamma} \times \frac{\gamma A}{AG} \text{ by § 7, part 2.}$$

§ 9. As in § 8, this is an obvious deduction from Ap. iii. 16-23.

§ 10. This is Desargues' Theorem: " If a quadrangle is inscribed in a conic, any transversal meets its three pairs of opposite sides and the conic in four pairs of points in involution.

§ 11. The parameter of a diameter AT being a third proportional to the diameter and its conjugate QQ', the parameter of $AT = \dfrac{CQ^2}{CA}$; $\therefore AB^2 = CQ^2$.

$$\therefore \text{ by Apollonius, i. 39, } DE^2 : CA^2 - CD^2 = CQ^2 : CA^2 = AB^2 : CA^2,$$

and by similar triangles, $\quad DF^2 : CD^2 \qquad\qquad = AB^2 : CA^2$;

$$\therefore DE^2 + DF^2 : CA^2 \qquad = AB^2 : CA^2;$$

$$\therefore DE^2 + DF^2 = AB^2.$$

It will be seen that the *Essais* consist of the enunciations, without proof, of some half-dozen propositions. G. Chrystal, in his article on Pascal in the *Encyc. Brit.* (11th ed.), speaks of them as a résumé of the larger work, which hardly seems an appropriate term, and he refers to a report of the *completum opus* by Leibnitz, who had seen it in Paris. Can anyone explain the relation between the two, or give us any further information respecting them ?

JOHN J. MILNE.]

227. The most promising sign in a boy is, I should say, Mathematics.— Lady Holland's *Memoirs of Sydney Smith* (Routledge), p. 241.

228. The supply of superstition has never failed, nor can the prophecies of mathematicians calculate its ultimate differentiation.—*Nation and Athenaeum*, April 14, 1923.

SUMMER SCHOOLS FOR ADVANCED MATHEMATICAL STUDY.

By Professor Sydney Chapman, M.A.

The following remarks are based on the assumption, which I believe to be a true one, that there is a desire for more advanced mathematical knowledge among many people in this country, such as those engaged in teaching mathematics in secondary schools, technological institutions, and colleges, as well as others not professionally associated with mathematics. At present the means of satisfying this desire, at least as regards organised courses of post-graduate study available for persons employed in teaching or otherwise, are almost entirely lacking in England. The most that is available, and that only in a few university centres, consists of occasional lectures given under the auspices of universities and such bodies as mathematical societies and branches of the Mathematical Association. Able and interesting as these lectures may be, the knowledge to be acquired from them must, in the nature of things, be somewhat disconnected and superficial. Their value lies in the introductory glimpse which they give of unfamiliar branches of study, or in their rapid survey of a wide field of knowledge. They are useful complements to more systematic study, but are not a substitute for it.

Only now is a beginning being made in the provision of regular courses of post-graduate mathematical lectures during the ordinary terms at the provincial universities (I speak in this respect of Manchester University, but I think the statement does not unjustly describe the position elsewhere). It is to be hoped that increasing use will be made of the facilities thus afforded. But at present the university studies of intending mathematical teachers are usually followed by a year of training in educational methods, involving what may be a necessary, but is certainly in general a complete and abrupt, break in the student's progress as a mathematician. Then follow two or three years of strenuous and exacting work while the teacher is preparing his own lesson courses, and learning, by actual experience, the technique of his profession. Afterwards, however, a time may come when he feels that his school work is well in hand and mapped out on a reasonably settled basis. He may then wish to devote some of his leisure and his spare intellectual energy to further progress in the knowledge of mathematical ideas and discovery. If so, he can, of course, and must read for himself, but to do this in isolation and entirely without guidance is to make less than the best possible use of his energies. The need at this point seems to be for a renewal and subsequent continuance of his association with the intellectual life of the universities. And if this were achieved it would, I think, be beneficial not only to the teacher himself, and through him to his pupils, but also to the universities.

The best method of bringing about this contact would seem to be by summer schools in agreeable surroundings. Summer schools for mathematical teachers are already held in this country, and have considerable vogue, but their purpose is different from that here suggested ; they address themselves primarily to the technical problems of school teaching. In America, however, a good deal of advanced mathematical teaching is given at various universities during the summer, and particularly at the great school of mathematical studies conducted during June, July, and August at the University of Chicago. Large numbers of teacher-students and others attend this school each year ; Chicago is fortunate in that the university and its surroundings are sufficiently attractive to enable study to be combined with agreeable holiday-making. The university session is there divided into four terms, one being in the summer. Attendance during this term ranks equally with that during any other term, and need not be consecutive, provided that the requisite courses of lectures are completed. Members of the university staff teach for three out of the

four terms in each year, and during the summer the staff is augmented by professors and lecturers from other universities. This plan has led to a development of higher studies in mathematics which, among teachers and other interested persons outside the universities, is without parallel in England.

This is not the place to discuss whether the English university system could or should be developed along similar lines ; these facts are recorded merely to illustrate the demand which has arisen elsewhere for facilities of the kind mentioned, in support of the suggestion that here also some provision should be made to meet the same need. As a beginning, perhaps the best way of organising such provision in the form of a summer school would be by committees appointed for the purpose by one or more branches of the Mathematical Association. This would at least ensure that those most immediately concerned would have the deciding voice as regards the general arrangements, the subjects to be studied, and the persons to be invited to lecture and act as directors of study. I would suggest that such a school should last for not less than a fortnight, and that various separate courses of study should be arranged, of which one only should be taken by any given student. The number of courses would naturally depend on the number attending the school, and on the size of staff which it could support. Each course might consist of about twenty lectures of one hour each (two being given daily, except on Saturdays) ; this, together with a moderate amount of daily reading, would enable a substantial advance to be made in each of the subjects during the fortnight, while leaving ample time for recreation. The financial side of the arrangements would probably not offer any insuperable difficulty. I do not know whether financial assistance for such a purpose could be obtained from public bodies, but, in any case, a tuition fee of about two pounds would probably cover organising expenses and provide moderate fees for the staff. The lectures would, of course, have to be specially prepared ; while being of post-graduate standard, they would need to assume less detailed recollection of the preceding work than is possible with post-graduate students who have not yet left the university.　　　　　　　　　SYDNEY CHAPMAN.

The University, Manchester.

All those interested in the formation of a Summer School of Mathematics, such as is indicated in the above article, are requested to communicate as soon as possible with Miss Garner, The Whalley Range High School, Manchester.

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229. Twenty years ago, the study of rational arithmetic was opposed and derided as a part of liberal education ; and the Government is now preparing to extend it even to the schools in which pauper children are taught.—*Athenaeum*, 1848, p. 38.

230. "This aspiring youth wanted to become a mathematician ; and he had heard that at the topmost summit of the mathematical tree stood a mysterious subject known as the doctrine of ' quantics,' a calculus of calculi, only to be grasped by the very furthest stretch of the abstract mathematical faculty. So he came and asked to be taught ' quantics.' It was in vain that Professor Sylvester suggested simpler preliminary geometrical and algebraical studies ; the young man wanted to learn ' quantics,' and nothing but ' quantics ' would he have."

231.　　　　　　　And Mrs. Sterling says odd things

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Gives lectures on elliptic springs.

—Praed, *Marriage Chimes*, p. 187.

232. Dante was free imagination—all wings—yet he wrote like Euclid.—Emerson, *Poetry and Imagination*, § Transcendency.

THE TEACHING OF MATHEMATICS TO TECHNICAL STUDENTS.

By L. B. BENNY, M.A.

THE Mathematical Association has done in the past, and continues to do, work of the highest importance in connexion with the teaching of Mathematics in Secondary Schools. Its activities have reacted with advantage on courses of instruction, methods of teaching, and inevitably also on examination syllabuses. *The Report on the Teaching of Geometry*, which is one of its most recent publications, is a valuable document, the influence of which may be far-reaching.

While studying this Report, I was driven to compare the state of mathematical teaching in Secondary Schools with that in Technical Institutions, and the comparison is very greatly to the disadvantage of the latter. No effective steps have been taken to consider the mathematical needs of technical students, and the most satisfactory way of dealing with them. The necessity for a Government Commission on Technical Education is now being urged from several quarters, and such a Commission, which is long overdue, is likely to be set up in the near future, but it will necessarily deal with general questions. The type of inquiry that I have in mind is one that must be conducted by teachers themselves, and a knowledge of the work done by the Mathematical Association in the past, leads me to suggest that the Association may feel able, in the future, to extend its activities to the realm of Technical Education.

I believe that teachers of mathematics in Technical Institutions are not very strongly represented in the Association—an obvious fault of the teachers, not of the Association. A knowledge of the Association and its work would, I feel sure, produce a considerable accession of strength on the Technical side, and it should not be difficult to take steps to secure this result.

The fundamental problems on the Technical side do not appear, in my opinion, to be so different from those involved in Secondary Schools, as to prevent the Association from forming accurate and valuable conclusions, even as it is at present constituted. The stages of mental development are the same for both classes of pupil—the difference is mainly in the nature of the teaching necessary, having regard to the particular object in view.

Technical students may be roughly divided into two grades. The first composes those who are taking courses of Degree standard, in most cases actually leading to a Degree in Engineering or some other branch of Technology. The course of mathematical study for students in this grade is fairly clearly defined, and offers no special difficulty. The second, and by far the largest grade, includes students taking courses distinctly below the standard of the first grade. It includes a large proportion of part-time students, who are aiming at qualifications such as the Ordinary and Higher National Certificates in Engineering granted by the Board of Education. Students of this type usually join the Institution at about fifteen years of age, and in the majority of cases have very little mathematical knowledge when they commence their course.

These students usually receive several years' instruction in "practical" mathematics. I have never been able to discover exactly what this is supposed to include; it appears to exclude, carefully, all the elements of mathematical instruction, which render it valuable as a mental training. The student often finishes by acquiring a certain degree of facility in applying mathematical processes to a number of problems in Engineering; of the inner meaning of these processes, and of the principles underlying them, he usually has not the faintest conception. The syllabuses of many of the Examining Bodies are

so framed, that the students can pass their tests, without having received any proper logical training in any one branch of mathematics.

To the objection, often raised, that students of this type cannot absorb any logical course of mathematics, in the time at their disposal, I would reply that no serious attempt has been made to construct such a course. I believe it should be possible to construct suitable courses, the theoretical framework of which would be necessarily shorter than for the Secondary schoolboy, but which would include all the theory necessary to enable the student to *understand* the applications he makes, not to make them by rule of thumb. His power of using his mathematics as a tool would be enormously increased in consequence.

The teaching given at present is lacking in uniformity, depending sometimes on the teacher (who is too often more of a practical man than a mathematical specialist), and sometimes on an Examination syllabus of the type I have mentioned.

It would add enormously to the efficiency of the instruction given, if courses could be devised in Algebra, Geometry, Trigonometry, and Calculus, which would aim at developing the subjects on their " useful " side, while preserving a reasonable minimum of logical theory. The general acceptance of such courses by Institutions and Teachers would rapidly result in a corresponding modification of the requirements of Examining Bodies.   L. B. BENNY.

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233. Letter from Dr. Thomas Chalmers to a Professor of Mathematics in one of the Scottish Universities :

Morningside, 7th September, 1844.

MY DEAR SIR,

Can you tell me of any author who treats of the properties and progression of prime numbers ? The following is a curious order, observed for some time, in the proportion which the composite numbers bear to all others, and from which I had hoped the absolute proportions of the composites to the primes throughout the whole infinity of numbers might have been ascertained within an indefinitely near approximation :

" The numbers in which 2 does not enter as an aliquot part are to number at large as 1 to 2, or $\frac{1}{2}$.

" The numbers in which 2 and 3 do not enter as aliquot parts are as 1 to 3 or $\frac{2}{3}$.

" The numbers in which 2, 3, 5 do not enter, as 4 to 15, or $\frac{4}{15}$.

" The numbers in which 2, 3, 5, 7 do not enter, as 8 to 35, or $\frac{8}{35}$.

" The numbers in which 2, 3, 5, 7, 11 do not enter, as 16 to 77, or $\frac{16}{77}$.

" See the promise then I had on entering this investigation, that, if you take the primes in order, 1, 2, 3, 5, 7, 11, etc., you would arrive at the general proposition that the composites formed of them successively would so run as to leave remainders, which have to all numbers proportions expressed by fractions, whose numerators each double its predecessor, as 2, 4, 8, 16, etc., and whose denominators were the products of the two last prime numbers that had been taken up in the progress of the investigation as $2 = 2 \times 1$, $6 = 2 \times 3$ $15 = 3 \times 5$, $35 = 5 \times 7$, and $77 = 7 \times 11$. Judge of my disappointment, then, when proceeding to the next prime number, 13, and expecting the result $\frac{32}{143}$. I found it very difficult, and thus has my goodly progression most cruelly been put an end to.—Yours very truly,   THOMAS CHALMERS."

Athenaeum, 1853, p. 670.

234. . . . I always end my letters by telling you (and the problems of Euclid are not more true) that I am your affectionate friend.—To Miss G. Harcourt. Sydney Smith, 1842.

235. My dear Lady Grey,— . . . When I am very nervous I always do sums in Arithmetic, and take camphor-julep. . . .—Sydney Smith, 1831.

MATHEMATICAL NOTES.

698. [**6. a.** β ; **9. b.**] *Change in Kinetic Energy when a Shell explodes (into two pieces), and when two Objects collide.*

Let the vector AB represent momentum MV of the whole (or of centre of mass).
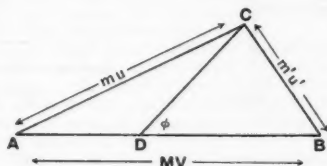
Let AC, CB represent momenta (mu, $m'u'$) of the parts.

Divide AB at D so that $AD : DB = m : m'$ or $\dfrac{1}{m'} : \dfrac{1}{m}$.

Then velocities of parts are $\dfrac{AC}{m}$ and $\dfrac{CB}{m'}$; and their relative velocity is $\left(\dfrac{1}{m} + \dfrac{1}{m'}\right) CD.$

Excess of K.E. of parts over that of the whole is

$$\frac{1}{2}\left(\frac{AC^2}{m} + \frac{CB^2}{m'} - \frac{AB^2}{M}\right)$$

$$= \frac{V}{2}\left\{\frac{AD^2 + DC^2 + 2AD \,.\, DC \cos \phi}{AD} + \frac{BD^2 + DC^2 - 2BD \,.\, DC \cos \phi}{DB} - AB\right\}$$

$$= \frac{V}{2}\frac{AD + DB}{AD.BD} \times DC^2 = \frac{M}{2mm'} \times DC^2$$

$$= \frac{M}{2mm'}\frac{(\text{rel. vel.})^2 \times (mm')^2}{M^2}$$

$$= \frac{1}{2} \times \frac{mm'}{M} \times (\text{rel. vel.})^2.$$

In the case of a bursting shell, this expression measures the increase of K.E. In the case of two bodies colliding, the relative velocity is reduced by the impact, and the loss of K.E. is

$$\frac{1}{2}\frac{mm'}{M} (\text{initial rel. vel.})^2 (1 - e^2).$$

The College, Malvern.   L. S. MILWARD.

699. [**K**^{1}. **5. a.**] *Gazette*, vol. xii. p. 20.

" Why did the lecturer miss this opportunity of improving my more cumbrous proof ? "   T. P. N.

Not, as it happens, for the reason suggested by Professor Nunn ; but because the lecturer had the uneasy feeling that to assume the Principle of Similarity was, *in effect*, to assume the *three* fundamental theorems on similar triangles.   This the lecturer was, *and is*, unwilling to do.   " WEE SLIP."
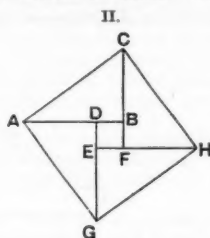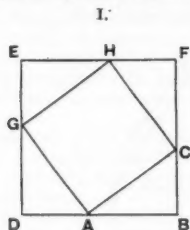
700. [**v.**]   Query.   Who first used $/x$ for $\log x$?   Was the use **ever** common ?   GALLINA.

**701.** [**K**¹. **3. c.**] *The Bridal Chair or Couch.*

Leslie (*El. of Geometry*, 1820) gives I. and II. in his notes on I. 47, and says that both " were long afterwards common among the Arabians and Persians, and by them communicated to the nations of India. The second

I.

II.

mode, however, would seem to be the favourite, since the figure used is in the Oriental languages styled *the bridal chair or couch*, alluding to its form or its prolific virtues." E. M. LANGLEY.

702. [**K**¹. **21. b.**] *P* is a point within a circle *ABC*. It is required to find a diameter *AEF* of the circle, such that *P* is a point of trisection of the arc *EPF* of the circum-circle of the triangle *EPF*. Draw radius *OB* perpendicular to diameter *COA* through *P*.
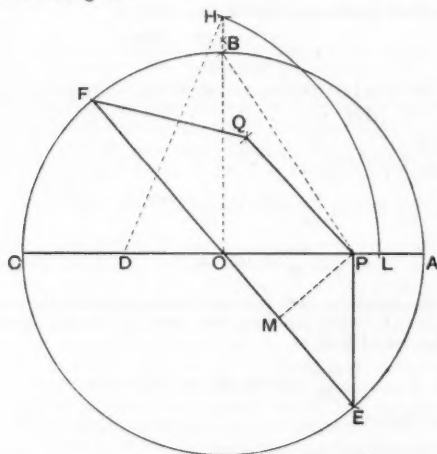
Bisect radius *CO* in *D*. Join *PB*. Take *OH* along *OB* equal to *PB*, *DL* along *DA* equal to *DH* and inflect *PE* to circumference equal *OL*. Then diameter *EF* through *E* is that required.

$$DL^2 = DO^2 + OL^2 + 2DO \cdot OL,$$
$$OP^2 + OB^2 + OD^2 = DO^2 + OL^2 + 2DO \cdot OL,$$
$$OE^2 + OP^2 = PE^2 + OE \cdot PE.$$
But
$$OE^2 + OP^2 = PE^2 + 2OE \cdot OM;$$
$$\therefore OM = \tfrac{1}{2}PE.$$

J. G. FURTADO.

REVIEWS.

Leçons sur les Fonctions Uniformes à point singulier essentiel isolé.
By G. Julia. Pp. vii + 150. 15 fr. 1924. (Gauthier-Villars.)

This book forms one of the well-known series of monographs edited by Borel. The high standard set by previous volumes is maintained, and the general aim of the series is achieved, since the work embodies in well connected form much fairly recent research, and leaves the reader with the feeling that the last chapters on the subject are still to be written.

In accordance with the Borel plan, the first chapter consists of a summary of results and methods which form the starting point or the basis of proof of the investigations in hand. The chief of these are here, conformal representation and the requisite properties of elliptic modular functions. The exposition of the latter is complete in itself, and requires no special knowledge. This is followed by a uniform treatment, the modular function forming the basis of proof throughout, of Picard's theorems on the exceptional values of integral functions, and of functions with an isolated essential singularity and of theorems by Landau and Schottky, which contain the former's first theorem as a special case. Goursat, *Cours d'Analyse*, vol. ii. (3rd ed., 1918), gives, in a note at the end, practically the same set of results, using a different line of proof.

For further developments the properties of, and criteria for, normal families of functions are necessary. The third chapter (pp. 52-88) is devoted to a full discussion of these. The method of their application consists in converting the study of a function in the neighbourhood of a point into that of a family of functions in some domain, usually a ring bounded by concentric circles, remote from the point. Before proceeding to his own applications, the author gives a summary of the results and methods of some researches by Montel, Lindelöf and Iversen.

The last three chapters deal mainly with the author's own contributions to the subject. By studying the points in which the families of functions considered are not normal he is able to go somewhat further than Picard: for example, if $f(z)$ is an integral function, he is able to show that it assumes every finite value save one at most, *for values of z restricted to a certain band of the plane*. The systematic use of the elliptic modular function and of normal families of functions naturally gives the work a quite different orientation from that of Borel's own monograph on integral functions.

One criticism we cannot refrain from making—the references are not as complete as one would like to have them. For example, the author evidently (p. 66) believes that a reference has been given for the journal in which Montel's work appeared, but it has not ; and later (p. 92), the phrase " M. Wiman a montré " can scarcely be regarded as the best help an eminent writer can give to a worker wishing to continue the subject further.

<div align="right">W. L. Ferrar.</div>

The Principle of Relativity. By H. A. Lorentz, A. Einstein, H. Minkowski and H. Weyl. With notes by A. Sommerfeld. Translated by W. Perrett and G. B. Jeffery. Pp. viii + 216. 12s. 6d. net. 1923. (Methuen.)

Relativity : A Systematic Treatment of Einstein's Theory. By J. Rice. Pp. xv + 397. 18s. net. 1923. (Longmans.)

Relativity and Modern Physics. By G. D. Birkhoff. Pp. xi + 283. 18s. 6d. net. (Harvard University Press and Oxford University Press.)

Prof. Jeffery and Dr. Perrett have rendered a great service to students of the theory of Relativity by giving a translation into English of eleven original papers by Einstein and others. These papers enable us to trace the origin of the theory and the continual changes during its development.

For many years physicists had tried to decide whether the motion of the earth had any effect upon the luminiferous ether. Astronomers explained aberration by taking the ether to be *stagnant* and quite unaffected by the

earth's motion. On the other hand, Fizeau's experiment on the velocity of light in flowing water seemed to indicate that the ether was partially carried along by the water. Michelson in 1881 thought that he had disproved the stagnant ether theory, but there was a slip in his calculations. When this had been rectified, the results were such as might have arisen only from errors of observation. Michelson (in collaboration with Morley) repeated his experiments with additional precautions, and arrived at the same conclusion as before. Lorentz (1895) tried to remove the contradiction by the hypothesis independently put forward by Fitzgerald that motion through the ether caused a solid body to contract in the direction of motion. This hypothesis was shown to account approximately for Michelson's result. Unfortunately this contraction appeared to imply the existence of other optical and electrical effects. Very careful search was made for these by Rayleigh and Brace, Trouton and Noble, and others, but nothing was found. Poincaré complained that apparently a special hypothesis had to be invented for each new experimental result.

In 1904, Lorentz put his previous work in a more complete form. He showed that all the experimental results could be accounted for, exactly and not merely approximately, provided that certain relations held between lengths, masses and electric and magnetic fields in a moving system and the corresponding quantities in a stationary system. It should be noticed that a sharp distinction was drawn between a stationary and a moving system. Contraction in a moving system was taken to be a real physical effect, due to the resistance of the ether. It was found convenient in the moving system to introduce a variable called the *local time*, which however was supposed to have no physical significance.

A year later Einstein (who had not then seen Lorentz's last paper) dealt with the same subject from quite a different point of view. He considered two systems K and K' moving uniformly relatively to each other, but these were treated on an equal footing. The "contraction" in the K' system, as estimated by an observer in K, was exactly paralleled by a "contraction" in the K system, as estimated by an observer in K'. The quantity which Lorentz called *local time* was now considered to be a real physical quantity which could be actually measured by light signals. In fact the whole theory was built up on two postulates asserting the equivalence for physical phenomena of the two systems, without any hypothesis concerning the ether. At a later date, Eddington asserted explicitly what seemed to be implied in Einstein's work, that nothing happened to the rod that Lorentz supposed to contract, and that the apparent contraction was merely due to the observer and the observed phenomena being in different systems.

Great efforts were made to extend the theory to non-uniform motion, and to deal with the relation between gravitation and light. Minkowski's geometrical treatment of uniform motion paved the way for the solution of the first problem. As for the second, considered apart from the first, Einstein in 1911 predicted that a ray of light passing near the sun would be deflected by a certain amount. However, in 1915 he saw that these two problems must be treated together, and the amount of deflection predicted was doubled. This prediction was strongly supported by the observations of the English eclipse expedition of 1919 and placed beyond all doubt by the American eclipse expedition of 1922. Another prediction made by Einstein in 1915, concerning the shift of lines in the solar spectrum, gave rise to much controversy. St. John declared that experiments disproved the fact, and many of Einstein's supporters doubted whether this prediction really was an essential part of the general theory. Einstein himself never wavered, and he declared that he was willing for his theory to stand or fall by the test of this matter. Other experimenters supported Einstein, and quite recently St. John has, after more careful researches, confirmed the result which he previously denied. The success of the theory in accounting for the motion of the perihelion of Mercury is too well-known to need mention.

In 1917 Einstein published his *Cosmological Theory*. He showed that the ordinary Newtonian ideas require that the heavenly bodies should be distributed round a sort of centre in such a way that the mean density of the matter of space should fall off more rapidly than the inverse square of the distance

from this point. But the energy in such a universe would waste away. Part of the radiation emitted by the heavenly bodies would stream outwards and be lost in the infinite, and from time to time the heavenly bodies themselves would be sent on a journey to infinity, whence they could never return. To escape this depressing conclusion, a hypothesis of a finite universe was advanced. This is difficult to understand, and it leads to startling conclusions for which there is no evidence at present. De Sitter's alternative theory appears to have some support from astronomical observations of the spiral nebulae, but there are many difficulties.

Although Einstein in 1915 dealt with electricity as well as gravitation, electricity was treated rather as something extraneous, which might or might not be present in the space whose properties were determined solely by the gravitational field. Weyl (1918) attempted to connect electricity also with the geometry of space. It is difficult to be sure how far Weyl now adheres to his original views. At any rate they are certainly not accepted by Einstein or Eddington, though in a modified form they have led to further progress. Eddington (1921) gave a generalisation of Weyl's geometry and Einstein (1923) used this to give a new and simpler method of linking up electricity and gravitation. This theory may be expected to develop in the near future.

The majority of the mathematical papers mentioned will be found in the collection under review, which can be warmly commended.

Mr. Rice professes to have written his *Relativity* primarily for the science undergraduate, for whom it does not appear very suitable, as it is very long and full of detail. If, however, it is looked upon as a work of reference, it has some good points not to be found elsewhere. Einstein's new electrical gravitational theory is included.

Prof. Birkoff's *Relativity and Modern Physics* is an outgrowth of a course of lectures given at Harvard University. The book appears to be specially adapted to American University conditions, which differ considerably from those in England. The experimental evidence is left very much in the background, and there is no mention of the importance of relativity ideas in quantum theory. On the other hand, the mathematical portions are treated more carefully than usual. H. T. H. PIAGGIO.

An Introduction to the Principles of Mechanics. By J. F. S. Ross. Pp. x + 400. 12s. net. 1923. (Jonathan Cape.)

This is a stimulating text-book, carefully thought out and written in an interesting manner. The author complains in the preface that much of the experimental work, on which reliance is now placed, seldom achieves the results intended. It is certainly true that elaborate experiments often defeat their own ends, but the sense of reality obtained through experiments of a simple type can scarcely be over-rated. Centre of Gravity, Coefficient of Friction and the Determination of the Value of "g" are, so far as the reviewer has noticed, the only subjects on which experimental work is mentioned, if we exclude Greenhill's experimental bicycle wheel in connection with gyroscopic motion. We believe that the author has gone too far in his self-denial. Moments, Uniform Acceleration, Periodic Motion and Moment of Inertia all lend themselves to simple experimental verification, in which the results obtained are correct to within a very small percentage error.

It is disappointing to come across in such a carefully written book the slovenly treatment of the Acceleration—common in Victorian text-books—in the time-honoured Atwood's Machine. We are told that :

"These two forces" (the respective weights of the two masses) "tend to move the string in opposite directions, and their resultant will therefore be $Mg - mg$, that is $(M - m)g$ poundals in the same direction as the greater force Mg. The net effective force acting upon the system is therefore $(M - m)g$ poundals, and the mass on which it acts is $M + m$ pounds, since both masses must move at the same time and to the same extent, and therefore with the same acceleration."

The natural inference to make from the former statement is that the Acceleration would be infinite since the string is weightless. The author is on safer ground in the latter sentence, when he considers the system consisting

of the string and the attached masses, but to apply the fundamental relation, Force = Mass × Acceleration, to the case of a non-rigid body, is at this stage quite unjustifiable : it would be necessary to go into a full discussion of the reasons why the reaction of the pulley and the tension of the string do not appear in the Equation for the Acceleration, and when all is said and done the result is obtained immediately by eliminating the Tension from the Equations of Motion for the separate Masses.

The author is particularly happy in his treatment of Normal Acceleration and Gyroscopic Motion. The analogies that he draws between them are novel and suggestive. The logical way in which the subject is developed, the clear type and the general arrangement are so good as to make the reading of the book a pleasure.

Dr. William Garnett has written a valuable introduction, in which he discusses the scientific training of the mechanical engineer. The instances that he adduces, of mistakes made by the practical man on the one hand and the scientifically trained engineer on the other, are just the sort of thing that appeals to every student of mechanics, whether he intends to become an engineer or is studying the subject as part of a general course in science.

Mechanics via the Calculus. By P. W. NORRIS and W. SEYMOUR LEGGE. Pp. xi + 340. 12s. 6d. net. 1923. (Longmans.)

The purpose of this book is to bridge over the gap that separates the elementary text-books from the more advanced treatises that are used in Honours Courses at the Universities. The subject matter includes Statics, Dynamics of a particle, Rigid Dynamics and Hydrostatics, and in addition there is a chapter on Shearing Force, Bending Moment and Deflection of Beams, which ought to prove useful to students of engineering. A special feature is the number of examples worked out in detail. These examples should be of great value to a student who has to depend on his own resources for the solution of new types of question. That the authors are teachers of experience is shown by these worked-out examples, which are chosen with much discrimination so as to cover as wide a field as possible. More than half the book is taken up with examples either illustrative or set for solution. The bookwork is reduced to a minimum, and in some of the chapters the student would find it advisable to have access to a text-book like Love's *Mechanics*, in which the foundations of the subject are laid with more firmness. The printing is good and the setting out of the worked examples is clear and methodical.   R. M. MILNE.

Notions sommaires de géométrie projective à l'usage des candidats à l'ècole polytechnique. Par M. D'OCAGNE. Pp. 24. 3 frs. 1924. (Gauthier-Villars.)

A very clear short treatment of the elements of homography and a few applications to conics and quadrics.

Cours complet de mathématiques spéciales. Tome IV. **Géométrie descriptive et trigonometrie.** Par J. HAAG. Pp. 152. 13 fr. **Exercices du tome IV.** Pp. 154. 15 fr. 1923. (Gauthier-Villars.)

The "descriptive geometry" of this volume includes nothing that would appear under that title in an English work. It deals in great detail with methods of drawing plans, elevations and apparent contours of the simpler solids and their intersections, with a chapter on maps and one on perspective, this being apparently the subject matter of a course at the École Polytechnique. The trigonometry consists of a couple of straightforward elementary chapters on circular functions and solution of triangles.

The book is designed for the kind of student who has to be told when to draw a line in full and when to dot it.

Y a-t-il continuité dans le monde physique? Par N. YERMOLOFF. Pp. 48. 3 fr. 50. 1923. (Dorn, Paris.)

By defining discontinuity so as to include any passage through zero (act of extinction), the author has no difficulty in answering his question in the negative.

Principles of Geometry. Vol. III. **Solid Geometry : quadrics, cubic curves in space, cubic surfaces.** By H. F. BAKER. Pp. 228. 15s. 1923. (Cambridge University Press.)

In his third volume, Prof. Baker continues the application of his methods to ordinary space. The logical treatment of the earlier volumes is taken for granted, which gives room for the fuller enjoyment of the " fascination and freedom " of the subject and the author's treatment of it.

The quadric is approached by means of its generators, and the discussion passes on very naturally to line coordinates and the linear complex. The usual metrical properties of quadrics and sphere are related to the theory of an absolute conic, corresponding to that of the absolute points of a plane explained in Vol. II.

The twisted cubic, " in many ways simpler than a conic," is introduced as the partial intersection of two quadric cones. This is somewhat artificial, as the full and very attractive treatment is based on the intersection of three related pencils of planes, and the consequent parametic representation of the curve.

For the cubic surface, the double six is the starting point, carrying on the emphasis on straight lines that marks the volume. The only singular cubic mentioned is that with four double points, and some properties are given ot its dual, Steiner's quartic surface. A short section on quartic curves is also included. H. P. H.

The Elements of Coordinate Geometry. Part II. **Trilinear Coordinates**, etc. By S. L. LONEY. Pp. viii + 228. 6s. 1923. (Macmillan.)

The first chapter of this volume deals with Cross Ratio, Homographic Ranges and Pencils, and Involution, and these subjects are continued in Chapter IV. Reciprocation and Projection occupy Chapters VI. and VII. A text-book containing a blend of Pure and Analytical methods accords well with modern ideas, but in the present volume the treatment of Pure Geometry is half-hearted. The postponement of Projection to so late a stage makes it unlikely that the student will appreciate the relevance of cross-ratio methods ; nor is he likely to be impressed by their power when he finds even such results as the harmonic property of the quadrilateral, the " cross-join " property of homographic ranges, and the involution theorem of Desargues and Sturm proved by analysis. No hint is given that this last theorem leads to many of the principal properties of the conic. If it is intended that Pure Geometry should be learnt from an independent treatise, then the space devoted to this part of the subject seems to be excessive.

Chapter II. introduces Trilinear coordinates and equations of the first degree. Results such as the formulae for the distance between two points, for the angle between two lines, and for the length of the perpendicular are proved by transference to Cartesian axes. Some of this work could be made simpler and more attractive by the use of elementary vector theory. Reasons exist for preferring Areals as the primary system, and other Homogeneous Coordinates deserve some mention.

Equations of the second degree are considered in the following chapter. The equation to the tangent is found, as in several text-books, by writing down the equation to the chord in a form slightly less convenient than

$$(aaa_1 + \ldots) + (aaa_2 + \ldots) = (aa_1a_2 + \ldots).$$

The process is rather artificial, and the method of Joachimstal, combined with a suitable notation, is surely preferable. After the properties of the general equation have been investigated, the equations to various circles and conics specially related to the triangle of reference are obtained ; envelope as well as locus equations are given, but their importance is not sufficiently emphasised. It is true that Chapter V. is devoted to Tangential Coordinates, but even at this stage the " principle of duality " has not been enunciated, and the treatment of the subject suffers accordingly. There is no need for details of metrical reciprocation, but the student must be made familiar with the idea of duality before he can be expected to acquire a grasp of envelope methods. The harmonic locus $\mathbf{F} = 0$ and the harmonic envelope $\Phi = 0$ are obtained, and might have afforded an excellent example of reciprocal analytical processes.

Some attention is paid to parametric representation, but the ordinary method of arriving at parametric equations is not given. Ex. 7 on page 149 is rather misleading, as it suggests that the representation is unique; the representation given for the conic $(x-y)^2 = z(x+y)$ is strangely complicated. In the closing chapter there is an account of the invariants of two conics, which does not, however, include consideration of Tangentials.

As in most text-books, there is no serious attempt to justify the use of complex numbers as coordinates, and the treatment of points at infinity is unsatisfactory.

The book is not disfigured by many misprints, but two of these occur in Ex. 24, p. 114, and Ex. 22 on the same page is stated incorrectly, as *seven* lines would be required to form the sides and diagonals of a quadrilateral. Elsewhere the "diagonal points" of a quadrangle are called "vertices." Ex. 8 on p. 129 is unintelligible. The number of examples exceeds 400, and solutions or hints are given for about 100 of them, with answers at the end of the book.   A. R.

Relativity and Gravitation : An Elementary Treatise upon Einstein's Theory. By T. Percy Nunn. Pp. 162. 6s. net. 1923. (University of London Press.)

A glance through Dr. Nunn's book (it was by no means a hurried glance,* but I cannot call it a reading) leaves me grateful to him for two things. Firstly, he makes but sparing use of imaginary coordinates, which seem to me unsuitable for dealing with fundamental physical conceptions. The mathematician, knowing that his processes give him correct results, does not worry himself about the interpretation of them, nor need he do so when talking to other mathematicians. But the expounders of the Theory of Relativity have done much talking to laymen, and they have been inconsiderate, in a considerable degree, in their language. We all know (we who read this *Gazette*) that a hyperbola $x^2 - y^2 = 1$ is only another form of a circle $x^2 + y^2 = 1$, but we need not confuse innocent people's thoughts by saying so to them. Moreover, it is by no means clear that we cannot think with profit ourselves a little more in terms of real things and less in riddles. There are, after all, pretty important differences between a circle and a hyperbola. The circle $x^2 + y^2 = 0$ is a point, the true symbol of a particle separated from other particles, even though it may attract them as Newton suggested. The hyperbola $c^2 t^2 - r^2 = 0$ is two straight lines stretching away into the infinite, symbolic of the newer conception of an event-particle linked up with vast numbers of other events past and future. To think of it as an imaginary circle or sphere cramps our ideas unnecessarily. Would not mathematicians themselves do well to drop the imaginary conversions at times—to drop the imaginary " rotation " and speak of the distortion which it really is, and to drop speaking of imaginary " time " $(t\sqrt{-1})$ as being a fourth dimension, returning to the actual fact that it is not " time " at all which is associated with space, but distance-travelled-by-light-or-gravitation, (ct), which is an actual distance related to actual space as we know it ? Much distressful bewilderment would have been saved if ct had been spoken of from the first as " travel," or by some name which shows what it really is, instead of by a name (" time ") which ignores its dimensions.

The other main point which pleases me in Dr. Nunn's book is that he leads up from the particular to the general, which is a much easier path for most of us than the descent from the general to the particular. I don't think that any one need be ashamed to admit this when such a man as Sir G. H. Darwin did not attempt to conceal it. Darwin was President of the R.A.S. for one year only (we pleaded for the usual two years, but he was adamant), but in that year we were lucky enough to award the Medal to Henri Poincaré, and consequently to obtain the views of one eminent worker on another's work in the same field. But that their methods were very different will be seen from the concluding paragraph of Darwin's Presidential address (*Mon. Not. R.A.S.*, Feb. 1900):

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* May I justify this plea by noting two trivial misprints—y for δy on p. 74, and δT for δT^2 on p. 85 ?

" The leading characteristic of M. Poincaré's work appears to me to be the immense wideness of the generalisations, so that the abundance of possible illustrations is sometimes almost bewildering. This power of grasping abstract principles is the mark of the intellect of the true mathematician ; but to one accustomed rather to deal with the concrete the difficulty of completely mastering the argument is sometimes great. To the latter class of mind the easier process is the consideration of some simple concrete case, and the subsequent ascent to the more general aspect of the problem. I fancy that M. Poincaré's mind must work in another groove than this, and that he finds it easier to consider first the wider issues from whence to descend to the more special instances. It is rare to possess this faculty in any high degree, and we cannot wonder that the possessor of it should have completed a noble heritage for the men of science of future generations."

I hope my readers will find as much pleasure in this passage as I have in reading it once again. To be frank, I had written what precedes it away from references (in the train) with but the general memories of nearly a quarter of a century ago to guide me, and it was only on return to Oxford that I found how completely Darwin had described Dr. Nunn's methods. (The whole address is well worth reading.) Tensors are put in the last chapter of the book, and the author does not even disdain to remind those of us who have accumulated a little rust of some of the more elementary processes in differentiating. The young man may regard the book with suspicion, as suggesting doubts of his ability to play first-class cricket ; and the old man may be already a little tired of the game. But there must be many who will enjoy playing in a comfortable way without strain, as Dr. Nunn has made possible.   H. H. TURNER.

Assurances sur la Vie. Calcul des Primes. By HENRI GALBRUN. Pp. 310. 35 fr. 1924. (Gauthier-Villars.)

It is inevitable as well as inadvisable for a text-book to contain much original matter, and the interest of any new book that comes within the wide term " text-book " lies in the point of view of the author. In a subject such as the calculation of premiums for life assurance the number of existing text-books is few and the possible points of view many. The best known books on the subject are the text-books on Life Contingencies issued by the Institute of Actuaries, the first by Mr. George King and the second by Mr. E. F. Spurgeon. In both these cases the point of view may be described as "utilitarian." Starting with the assumption that we know the rates of mortality that will prevail, the subject is developed on its practical side, and the mathematics involved are used merely to reach practical results and do not attempt to find a point of contact with the mathematical developments of the theory of probability that now take so prominent a place in modern statistical treatment.

In the book before us Dr. Henri Galbrun approaches the subject from the theory of probability, and discusses at frequent intervals the application of the law of great numbers and the deviations from the expected result. This is interesting ; but one cannot help feeling that the treatment does not take us very far : and the analysis when it has to be used statistically has to be simplified down to such an extent that we almost lose sight of the mathematical analysis.

The book starts with a general discussion of interest and the application of the theory of probability to a few simple examples that might occur in connection with insurance, and then follows the inevitable reference to Bernouilli and Tchebicheff. We then have a short discussion of tables of mortality with a reference to the Gompertz-Makehan law of mortality and to Woolhouse's method of graduation and a discussion of the errors in a graduated and ungraduated table respectively. The third chapter discusses interpolation, summation and approximate integration, and then in Chapters IV. to VIII. we have a treatment of life annuities, premiums for assurances, etc. The evaluation of the ordinary formulae follows the lines with which readers have become familiar, but in dealing with the numerical work, Dr. Galbrun seems to favour Lubbock's formula rather than the application of Simpson's rule or of some other quadrature formula.

Although we think it probable that the development of the subject will lie in applying it to the modern developments of the theory of probability, we are not in every instance satisfied with the methods suggested or the approximations used. Incidentally we may express a doubt whether it will ever be practicable to load the premiums in exceptional cases or in small classes of risk to allow for the deviations that may arise in such classes.

It is a little difficult to " place " Dr. Galbrun's book ; it seems to us more likely to appeal to the mathematician or mathematical statistician than to the practising actuary ; but the author would naturally have in mind the continental actuary rather than the English actuary, and the former is more " mathematical " and less closely identified with practical business than the latter. W. PALIN ELDERTON.

CORRESPONDENCE.

To the Editor of the *Mathematical Gazette.*

ETON COLLEGE, WINDSOR.

Dear Sir,

If any of your readers would care to borrow the graphs or the model of a frequency-surface which I showed at the January meeting, or the drawings from which a carpenter can make such a model, I shall be exceedingly glad to lend them.—Yours faithfully, W. HOPE-JONES.

ERRATA.

xi. p. 22, l. 1, *for* " of this volume " *read* " vol. x."

xi. p. 313, l. 1, *for* " coincoid " *read* " conicoid."

xi. p. 384, l. 6 *up, for* " 677 " *read* " 678."

xi. p. 427, *for line* 13 *up read :*

" then one in $9n - 8$ of his answers which satisfy the 9-test will be wrong."

THE LIBRARY.

160 CASTLE HILL, READING.

ADDITIONS.

The Librarian reports gifts as follows :

Sir THOMAS HEATH has presented a fitting and generous memorial of his tenure of office as President of the Association, in copies of his

The Thirteen Books of Euclid's Elements (3 vols).	. . . . . .	1908
A History of Greek Mathematics (2 vols.)	. . . .	1921

From Mr. T. W. HOPE :

A. L. BOWLEY	Elementary Manual of Statistics	1910
D. B. MAIR	School Course of Mathematics	1907

Only one reply was possible to a letter from Prof. S. L. LONEY beginning : " In reading the . . . *Gazette* of Dec. 1923 . . . I see . . . that there is no copy of my *Coordinate Geometry* in the Library of the Association. If you would like to have a copy I should be most pleased to have one sent. Are there any others of my books which you have not got and which you would care to have ?

If so, I shall be pleased to have them sent also." The sequel was the generous gift of new copies of the following :

Coordinate Geometry	1919
Solutions of Examples in ... *Coordinate Geometry* (By A. S. Gosset-Tanner and S. L. Loney)	1922
Dynamics of a Particle and of Rigid Bodies	1923
Elementary Dynamics	1921
Solutions of Examples in ... *Elementary Dynamics*	1909
Elements of Hydrostatics	1923
Solutions of Examples in *Elements of Hydrostatics*	1912
Solutions of Examples in *Plane Trigonometry, Part I.*	1921
Solutions of Examples in *Plane Trigonometry, Part II.*	1912
Statics	1920
Solutions of Examples in *Elements of Statics and Dynamics*	1923
Elements of Trigonometry	1920

From Rev. J. J. MILNE :

ARCHIMEDES	Method. Ed. and trans. into English by T. L. Heath	1912
P. BARLOW	Theory of Numbers	1811
M. BLAND	Algebraical Problems	1841
C. BOSSUT	General History of Mathematics. Trans. (from French into English) by T. O. Churchill	1803
F(RERE), I. C.	Eléments de Mécanique	1881
M. CHASLES	Géométrie Supérieure	1852
A. C. CLAPIN	Optical Problems	1850
J. S. CLARKE	Properties of the Parabola Proved Geometrically	1852
T. CRAIG	Motion of a Solid in a Fluid, and Vibrations of Liquid Spheroids	1879
L. CREMONA	Géométrie Projective. Trans. (from Italian into French) by E. Dewulf	1875
	The Library has this book in German also, but neither in English nor in the original Italian!	
J. EDALJI	Reciprocal Polygons	1898
M. GARDINER	The 'Three Sections', the 'Tangencies', and a 'Loci Problem' of Apollonius.	1860
T. GASKIN	Solutions of Geometrical Problems	1847
D. GAUTIER.	Hyperboles Etoilées	1911
G. L. HOUÉL	Commentatio de Coni Scaleni Proprietatibus	1833
	A Gröningen dissertation.	
C. HUTTON	Mathematical and Philosophical Dictionary (2 vols.)	1796
S. F. LACROIX	Elémens de Géométrie	1819
D. LARDNER	Plane and Spherical Trigonometry	1828
J. LESLIE	Geometrical Analysis, and Geometry of Curve Lines	1821
	This is Vol. 2 of a Course : Vol. 1 would be welcome.	
G. B. MATHEWS	Theory of Numbers. Part I.	1892
	The projected continuation was never published.	
J. J. MILNE	Companion to Weekly Problem Papers	1888
	Solutions of Weekly Problem Papers	1905
	It was in response to the Librarian's request that Mr. Milne's gift included copies of such of his own works as were lacking.	
A. MOREL	Géométrie Elémentaire des Sections Coniques	1891
G. PEACOCK	Algebra (2 vols.)	1842, 1845
S. P. RIGAUD and S. J. RIGAUD	Correspondence of Scientific Men of the Seventeenth Century (2 vols.)	1841
	Can any member give the Index to these volumes, which was published in 1862 ?	
I. TODHUNTER	Differential Calculus	1864
	Integral Calculus	1862
R, TOWNSEND	Modern Geometry of the Point, Line, and Circle (2 vols. in one)	1863, 1865

B. WILLIAMSON	•	Differential Calculus	-	-	-	-	-	-	-	1873
		Integral Calculus	-	-	-	-	-	-	-	1877
R. WILSON		Plane and Spherical Trigonometry	-		-	-	-	-	1831
J. WOOD		Algebra. Revised, etc., by T. Lund		-	-	-	-	1841
J. M. F. WRIGHT		Theory of Numbers	-	-	-	-	-	-	-	1831

From Mr. H. PEAT :

| J. HYMERS | | Conic Sections | - | - | - | - | - | - | - | - | 1845 |

It will be noticed that two of these gifts were made to repair losses notified in the *Gazette* for December last ; readers are asked to bear in mind the list of losses printed there, and the list of gaps printed in the preceding number.

PAMPHLETS. Mr. Milne's gift included offprints of papers by R. C. Archibald, R. F. Davis, and A. Lodge, and a booklet of portraits and facsimile pages arranged for Ginn and Co. by D. E. Smith. Prof. F. Cajori and Miss H. P. Hudson have sent copies of papers of their own.

REVUE SEMESTRIELLE DES PUBLICATIONS MATHEMATIQUES.

This journal abstracts the mathematical literature of the world. Current numbers have for some time been received in exchange for the *Gazette* ; recently a second-hand set of the early volumes was available, and this having been purchased, the Library has now a set complete from the beginning in 1893.

The merits of this *Revue* were the subject of a note by the late P. E. B. Jourdain in the *Gazette* for October, 1916 (vol. 8, p. 312).

The following books have been bought ; they are from Prof. Forsyth's library, and were offered at a low price by the buyer of the lots in which they were included :

H. F. BAKER		Abelian Functions	-	-	-	-	-	-	-	1897
A. R. FORSYTH		Theory of Functions	-	-	-	-	-	-	1900
G. H. HARDY		Pure Mathematics	-	-	-	-	-	-	1908

A REQUEST.

The Librarian hopes soon to attempt systematically to repair gaps in the runs of journals. The matter is of the utmost importance ; every investigator will agree that few experiences are more tantalising than to run an out-of-the-way periodical to earth only to find the volumes imperfect. To publish a list of gaps would be expensive and unprofitable, but the Librarian would urge every member who at any time had odd numbers of journals from the crowded shelves in London to make sure that he is not unwittingly giving them hospitality that they no longer need.

FOR SALE.

BOOLE. Calculus of Finite Differences. 2nd ed. 1872. In excellent condition. £1 7s. 0d. Apply Box 1, *Math. Gazette.*

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236. (Lamb) . . . abused me [B. R. Haydon] for putting Newton's head into my picture,—" a fellow," said he, " who believed nothing unless it was clear as three sides of a triangle." And then he and Keats agreed that he had destroyed all the poetry of the rainbow, by reducing it to the prismatic colours. It was impossible to resist him, and we all drank " Newton's health and confusion to mathematics."—*Life of B. R. Haydon,* by Tom Taylor (1853).

BOOKS RECEIVED, CONTENTS OF JOURNALS, ETC.

March, 1924.

Interlingua. By G. Peano. Pp. 16. n.p. 1923. (Cavoretto—Turin.)

Y a-t-il Continuité dans le Monde Physique? By N. Yermoloff. Pp. 48. n.p. 1923. (O. & G. Dorn, Paris.)

Notions Sommaires de Géométrie Projective. By M. Ocagne. Pp. 25. 3 fr. 1923. (Gauthier-Villars.)

The Development of the Sciences : Lectures at Yale University. Edited by L. L. Woodruff. Pp. xiv + 327. 16s. net. 1923. (Per Ox. Univ. Press.)

Four-Figure Mathematical Tables. By F. Castle. Pp. 48. 1s. 1923. (Macmillan.)

Elementi di Aritmetica, con note storiche e numerose questioni varie, per le scuole medie superiori. 6th ed. By G. Fazzari. Pp. 196. 7l. 1923. (Trimarchi, Palermo.)

An Introduction to the Study of Alternating Currents. By A. E. Clayton. Pp. vi + 296. 10s. 6d. 1923. (Longmans, Green.)

Engineering Mathematics. Part I. By R. W. M. Gibbs. Pp. iv + 64 + iv. 1s. 6d. 1923. (Blackie.)

Technical Arithmetic. By R W. M. Gibbs. Pp. viii + 168. 3s. 6d. net. 1923. (Blackie.)

The Psychology of Algebra. By E. L. Thorndike and others. Pp. xi + 483. 1923. (Macmillan Co.)

Plane and Solid Geometry. By W. B. Ford and C. Ammerman. Edited by E. R. Hedrick. Pp. x + 356 + xxviii. 1923. (Macmillan Co.)

Einleitung in die Mengenlehre. By A. Fraenkel. 2nd edition. Pp. ix + 249. 2·60 dollars. 1923. (Springer, Berlin.)

Elementary Mathematical Astronomy. By C. W. C. Barlow and G. H. Bryan. Pp. xvi + 445. 9s. 6d. net. 1923. (Univ. Tutorial Press.)

Unconventional Arithmetical Examples for Junior Scholarships. Teachers' Edition. By R. S. Williamson. Pp. vii + 70. n.p. 1923. (Cam. Univ. Press.)

The New Physics. Lectures for Laymen and Others. By A. Haas. Translated by R. W. Lawson. Pp. ix + 165. 6s. net. (Methuen.)

Our Debt to Greece and Rome. Mathematics. By D. E. Smith. Pp. x + 175. 5s. net. 1923. (Harrap.)

Mechanics via the Calculus. By P. W. Norris and W. S. Legge. Pp. xi + 340. 12s. 6d. net. 1923. (Longmans.)

Principles of Geometry. Vol. III. *Solid Geometry. Quadrics. Cubic Curves in Space. Cubic Surfaces.* By H. F. Baker. Pp. xix + 228. 15s. net. 1923. (Cambridge Univ. Press.)

Algebras and their Arithmetics. By L. E. Dickson. Pp. xii + 241. 2.25$. 1923. (Univ. Chicago Press.)

Publications de la Faculté des Sci. de l'Université Masaryk. 1923. (Pisa, Brno, Ceská 28.) *Les variétés à Deux Dimensions dans l'Espace à Quatre Dimensions.* Pp. 22. V. Hlavaty. *Sur les Racines Imaginaires de l'Equation T (z) = a.* Pp. 27. O. Borůvka. *Courbes tracées sur une Surface dans d'Espace affine.* Pp. 47. E. Čech. *L'Équilibre de l'Electricité sur une Surface Cylindrique.* Pp. 14. B. Hostinsky. *Contribution à la Théorie des Équations aux Différences finies.* Pp. 9. J. Kaucký.

Abhandlungen aus dem Mathematischen Seminars der Hamburgischen Universität. (Math. Seminars, Hamburg.)

Band II. 1923.

Die Natürliche Geometrie. Pp. 1-36. J. HJELMSLEV. *Über Analysis Situs.* Pp. 37-68. H. TIETZE. *Über affine Geometrie,* xxxviii., xxxix. Pp. 69-73. T. BIEHL. *Andwendung der Geometrie der Zahlen auf die indefiniten ternären quadratischen Formen.* Pp. 74-80. M. FUJI-WARA. *Über analytische Funktionen und algebraische Zahlen.* Pp. 81-111. H. BEHNKE.

American Journal of Mathematics. (Johns Hopkins Press, Baltimore, Ind.)

April, 1923.

A Class of Numbers connected with Partitions. Pp. 73-82. E. T. BELL. *Note on a New Type of Summability.* Pp. 83-86. N. WIENER. *On Mediate Cardinals.* Pp. 87-92. D. WRINCH. *Periodic Oscillations of Three Finite Masses about the Lagrangian Circular Solutions.* Pp. 93-121. H. E. BUCHANAN. *On Certain Chains of Theorems in Reflexive Geometry.* Pp. 122-144. F. D. SUTTON. *A Poristic System of Equations.* Pp. 145-153. L. B. ROBINSON.

July, 1923.

Systems of two Linear Integral Equations with two Parameters and Symmetrizable Kernels. Pp. 155-185. M. BUCHANAN. *The Asymptotic Expansion of the Function* $Wk, m(z)$ *of Whittaker.* Pp. 186-191. F. W. MURRAY. *Some Geometric Applications of Symmetric Substitution Groups.* Pp. 192-207. A. EMCH. *On Elliptic Cylinder Functions of the Second Kind.* Pp. 208-221. S. DHAR.

The American Mathematical Monthly. (Lancaster, Pa.)

Nov. 1923.

The Development of "Partitio Numerorum," with particular reference to the work of Messrs. Hardy, Littlewood and Ramanujan. Pp. 354-369 (to be continued). A. J. KEMPNER. *A contribution of Leibniz to the History of Complex Numbers.* Pp. 369-374. R. B. M'CLENON. *Vector Analysis of a Surface.* Pp. 374-382. J. B. REYNOLDS. Discussions :—*In what quadratic realms of rationality, and for what values of the prime integer* p, *is the function* $(x^p-1)/(x-1)$ *factorable ?* Pp. 382-384. L. WEISNER. *The Infinite and Imaginary in Algebra and Geometry.* Pp. 384-391. W. L. G. WILLIAMS.

Annali di Matematica Pura ed Applicata. (Zanichelli, Bologna.)

S. IV. Vol. I. Nov. 1923.

Nuova trattazione della geometria proiettivo-differenziale delle curve sghembe. Pp. 1-18. G. SANNIA. *Sui fondamenti logici della Mathematica secondo le recente vedute di Hilbert.* Pp. 19-29. M. CIPOLLA. *Sul moto sferico vorticoso di un fluido incompressibile.* Pp. 31-55. B. SEGRE. *Sulla definizione dell' integrale.* Pp. 57-82. B. LEVI. *Sulle soluzioni non analitiche dell' equazione funzionale* $f(x^2)-[f(x)]^2 = Kx$ *e su quelle analitiche dell' equazione* $f(x^2)=\lambda[f(x)]^b$. Pp. 83-84 (to be continued). G. ANDREOLI.

Bolletino della Unione Matematica Italiana. (Zanichelli, Bologna.)

Dec. 1923.

Sui determinanti ortogonali. Pp. 161-163. O. NICOLETTI. *Applicazioni geometriche di una formula di F. Siacci.* Pp. 167-170. F. SIBIRANI. *Sopra alcuni sviluppi in serie di funzioni fundamentali.* Pp. 173-176. C. SEVERINI.

Bulletin of the American Mathematical Society. (Lancaster, Pa.)

Nov. 1923.

An Electromagnetic Theory of Light-Darts. Pp. 385-393. H. BATEMAN. *Some Left Co-Set and Right Co-Set Multipliers for any given Finite Group.* Pp. 394-398. G. A. MILLER. *The Second Mean Value Theorem for Summable Functions.* Pp. 399-400. M. B. PORTER. *Analogies between the* u_n, v_n *of Lucas and Elliptic Functions.* Pp. 401-406. E. T. BELL. *Singularities of Curves of given Order.* Pp. 407-414. T. B. HOLLCROFT.

Dec. 1923.

An Introductory Account of the Arithmetical Theory of Algebraic Numbers and its Recent Developments. Pp. 445-463. L. J. MORDELL. *Integral Solutions of* $x^2-my^2=zu$. Pp. 464-467. L. E. DICKSON. *On the Reality of the Zeros of a* λ-*Determinant.* Pp. 467-469. R. G. D. RICHARDSON.

L'Enseignement Mathématique. (Gauthier-Villars.)

1923. Nos. 1-2.

Méthodes d'approximation dans le calcul du nombre des points à coordonnées entières. Pp. 5-29. J. G. VAN DER CORPUT. *Application des méthodes de H. Grassmann à la géométrie métrique du plan.* Pp. 30-89. P. C. DELENS. *Application de l'integration par. parties au développement en série.* Pp. 90-→.

Gazeta Matematica. (Chibrituri, Bucharest.)

Sept. 1923.

O Problema si o Teorema asupra Indicatorilor. Pp. 16-19.   L. LINTES.

Oct. 1923.

Simeon Marcovici. Pp. 41-43. TR. LALESCU. *Asupra cronografului Joly.* Pp. 43-47. LOCOT. I. LINTES.

Nov. 1926.

O Chiestiune de Geometrie Analitica. Pp. 81-82. E. ABASON. *Proprietati ale unui Triunghiu important.* Pp. 82-84. C. I. GEORGESCU.

Dec. 1923.

Rezolvarea unui sistem de ecuații. Pp. 123-129. N. KAUFMANN.

Jan. 1924.

Teoreme asupra triunghiului. Pp. 161-166. G. C. MOISIL. *O generalizare a teoremei lui Franke.* Pp. 167-169. M. I. FOCSENEANU.

Intermédiaire des Mathématiciens. (Gauthier-Villars.)

July-Aug. 1923.

Jahresberichte d. Deutschen Mathem.-Vereinigung.

XXXII. 1. Abt. Heft 1/4.

Arthur Schoenflies. Pp. 1-6. L. BIEBERBACH. *Zum Gedächtnis an H. A. Schwarz.* Pp. 6-13. GG. HAMEL. *Paul Stäckels Verdienste um die Gesamtausgabe der Werke von Leonhard Euler.* Pp. 13-32. F. RUDIO. *Über die Multiplizität der Schnittpunkte von zwei algebraischen Kurven.* Pp. 32-42. H. KAPFERER. *Neuere Untersuchungen in der Theorie der divergenten Reihen.* Pp. 43-67. K. KNOPP. *Über Vektordivision.* Pp. 67-86. M. WINKELMANN. *Vektorielle Begründung der Sphärischen Trigonometrie.* Pp. 86-91. K. KOMMERELL. *Der Ricci-Kalkül.* Pp. 91-96. J. A. SCHOUTEN.

The Journal of the Indian Mathematical Society   (Varadachari, Madras.)

Aug. 1923.

On a General Theorem relating to the Product of two Determinants. Pp. 73-90. C. KRISHNAMACHARI and M. B. RAO. *Easy Group Theory.* Pp. 91-96. G. A. MILLER.

Journal of the Mathematical Association of Japan for Secondary Education.

July, 1923.

Mathematics Teacher. (Camp Hill, Pa.)

Nov. 1923.

Mathematics Club Program. Pp. 285-390. A. H. WHEELER. *Craig's Edition of Euclid : Its " use and application " of the principal propositions given.* Pp. 391-397. A. G. ROWLANDS. *The Origin of our Numerals.* Pp. 398-401. C. P. SHERMAN. *Live Problem Material in Algebra.* Pp. 402-413. D. S. DAVIS. *Measuring achievement in First Year Algebra.* Pp. 414-420. H. R. DOUGLASS. *Advantages of a General Course in Mathematics for the first two years in High School.* Pp. 421-424. L. A. M°COY. *On the Precedence of Numerical Operations.* Pp. 425-430. R. L. MORITZ. *The Place of the Calculus in the Training of the High School Teacher.* Pp. 431-439. B. COSBY.

Nieuw Archief voor Wiskunde. (Noordhoff, Groningen.)

XIV. 2.

Over de Vertikale Projectie van de op het Oppervlak van een rechten Cirkelkegel met Vertikale as gelegen Kromme, die bij ontwikkeling ven del Kegelmantel een Cirkel wordt. Pp. 89-101. H. J. E. BETH. *Over de Gelijkheid van Herhaalde en Meervoudige Limieten.* Pp. 102-121. J. RIDDER. *De Indicator van Euler.* Pp. 122-131. N. R. PEKELHARING. *Over het Osculatievlak.* Pp. 132-136. J. WOLFF. *Over een Stelling van Abel.* Pp. 137-141. H. BREMEKAMP. *Vraagstukken der Variatierekening in de Leer der Frequenties.* Pp. 142-152. W. P. THIJSEN. *Over het Osculatievlak.* Pp. 153-158. H. B. A. BOEKWINKEL.

Periodico di Matematiche. (Zanichelli, Bologna.)

Nov. 1923.

L'attuale modelle di atomo. Pp. 381-394. E. PERSICO. *La teoria della relatività nella fisica dell' atomo.* Pp. 394-407. G. FURIANI. *I trattati di calcolo infinitesimale de Newton a Cauchy.* Pp. 408-427. I. ZAVAGNA. *Un' osservazione relativa ai problemi di 3° grado.* Pp. 428-429. C. ROSATI. *Ancora su la " Regola di Torricelli."* Pp. 429-430. E. BORTOLOTTI.

Revista de Matemáticas y Fisicas Elementales. (Calle Perú 147, ix. Buenos Aires.)

Aug. 1923.

Las Ecuaciones del Punto, de la Recta y del Plano. Pp. 73-75. B. I. BAIDAFF. *El " Multiplicator Perfettus."* Pp. 75-80. J. BABINI.

Sept. 1923.

Divisibilidad de relaciones geométricas. Pp. 97-99. TR. A. DORDEA. *Construcción de Cónicas.* Pp. 99-101. J. S. CORTI.

Oct. 1923.

Notas sobre las Progresiones. Pp 121-125. A. E. DE CESARE. *Relaciones métricas en un Triángulo cuyos lados están en Progresión Geométrica.* Pp. 126-129. A. MICLESCU.

Nov. 1923.

Progresiones geométricas con dos, tres y más razones. Pp. 145-150. B. IG. BAIDAFF.

Revista Matemática Hispano-Americana. (Soc. Mat. Española, Madrid.)

Sept. 1923.

Análisis Situs Combinaris. Pp. 209-218. H. WEYL. *Aplicaciones Matemáticas a la Aerodinámica.* Pp. 219-224. E. HERRERA.

Oct., 1923.

Análisis Situs Combinatorio (continued). Pp. 241-249. H. WEYL. *Sobre los Valores de las Funciones Enteras.* Pp. 250-253. P. PINEDA.

Nov.-Dec. 1923.

Análisis Situs Combinatorio (cont.). Pp. 273-279. H. WEYL. *Sobre los valores de las Funciones Enteras* (concluded). Pp. 280-285. P. PINEDA.

Revue Semestrielle. (Gauthier-Villars.) Vol. XXX. Oct. 1922-April 1923. Pp. 133.

School Science and Mathematics. (Smith & Turton, Mount Morris, Illinois.)

June, 1923.

The Rolling Cylinder and the Work Equation. Pp. 536-538. H. W. EDWARDS. *The Evolution of our Exponential Notations.* Pp. 573-581. F. CAJORI. *Socrates teaches Mathematics.* Pp. 581-584. N. ANNING.

Oct. 1923.

Organisation of Secondary Mathematics. Pp. 638-647. W. H. HART. *Mathematical Concepts in Current Literature.* Pp. 648-655. F. C. TOUTON.

Nov. 1923.

The Relation between Thinking and Memorization in Mathematics. Pp. 760-770. E. W. ATKINS.

Dec. 1923.

Preparation of Teachers of Mathematics for Junior High Schools. Pp. 842-852. J. R. OVERMAN. *Mathematical Errors sanctioned by Modern Usage.* Pp. 853-855. G. A. MILLER.

Jan. 1924.

Stereoscopic Harmonic Curves. Pp. 29-36. W. F. RIGGE. *A Thread of Mathematical History and some Lessons.* Pp. 37-45. Z. FERGUSON. *Geometry a Laboratory Science.* Pp. 58-71. W. A. AUSTIN.

The School Science Review. (Murray.)

Aug. 1923.

Generalizarea unei probleme de Aritmetica. Pp. 441-447. GH. BUICLIU.

Sept. 1923.

The Hodograph in Elementary Dynamics. Pp. 31-33. F. E. HACKETT.

Unterrichtsblätter für Mathematik und Naturwissenschaften. (O. Salle, Berlin.)

XXIX. 3 and 4.

Forschung und Schule. Pp. 34-37. D. MANKE. *Die Quantentheorie.* Pp. 37-40. O. HESS.

5 and 6. 1923.

Zur älteren Geschichte der Nomographie. Pp. 54-59. P. LUCKEY.

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Its purpose is to form a strong combination of all persons who are interested in promoting good methods of teaching mathematics. The Association has already been largely successful in this direction. It has become a recognised authority in its own department, and has exerted an important influence on methods of examination.

The Annual Meeting of the Association is held in January. Other Meetings are held when desired. At these Meetings papers on elementary mathematics are read and discussed.

Branches of the Association have been formed in London, Southampton, Bangor, Yorkshire, Bristol, Manchester, Cardiff, Sydney (New South Wales), and Queensland (Brisbane). Further information concerning these branches can be obtained from the Honorary Secretaries of the Association.

"**The Mathematical Gazette**" (published by Messrs. G. BELL & SONS, LTD.) is the organ of the Association. It is issued at least six times a year. The price per copy (to non-members) is usually 2s. 6d. each. The *Gazette* contains—

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(2) NOTES, generally with reference to shorter and more elegant methods than those in current text-books;

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(4) SHORT NOTICES of books not specially dealt with in the REVIEWS;

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